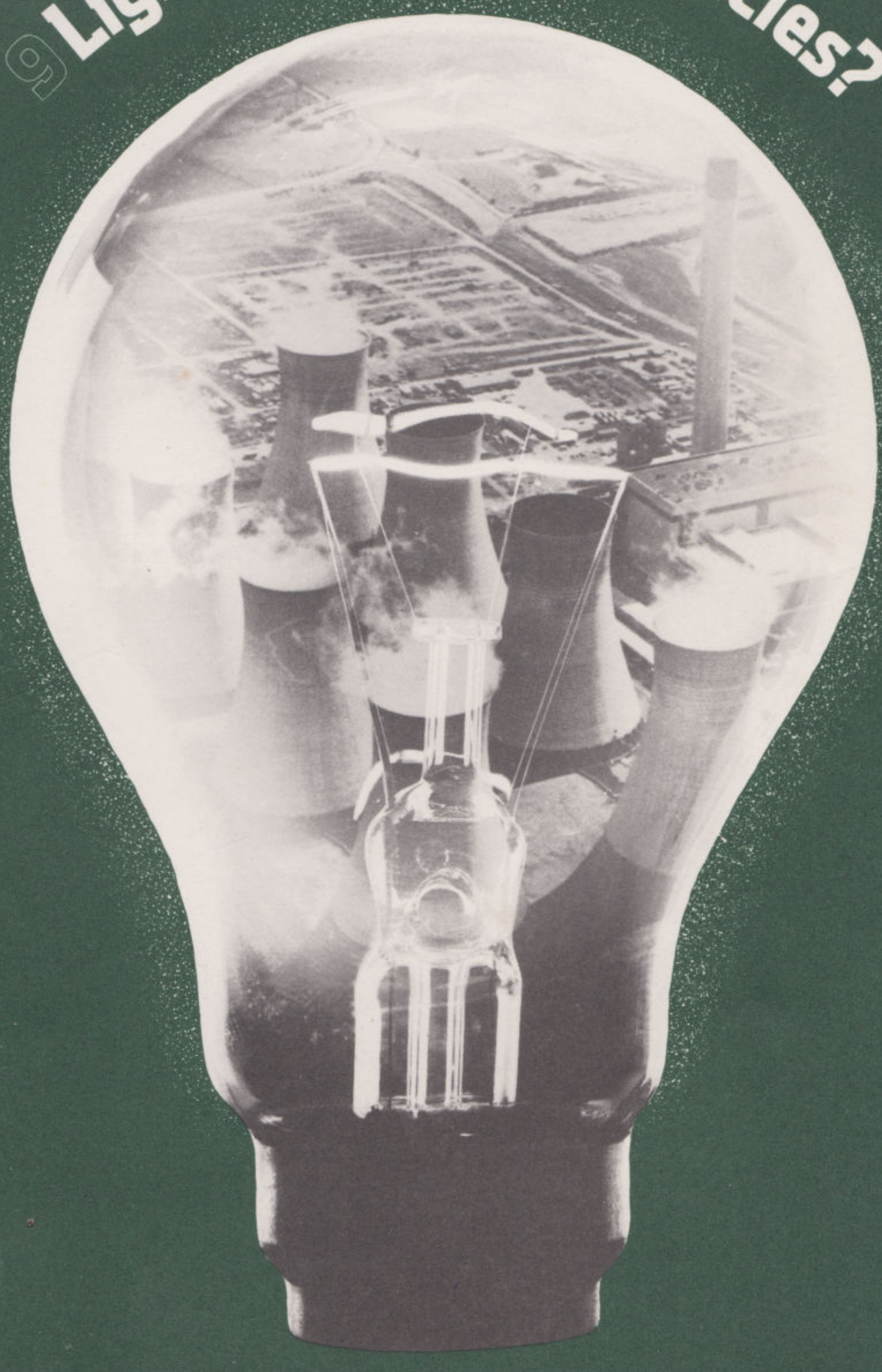
S101 Units 8 and 9

THE OPEN UNIVERSITY

Science: A Foundation Course

8 Energy

9 Light: Waves or Particles?



The Open University

Science: A Foundation Course

Unit 9
Light: waves or particles?

Prepared by the Science Foundation Course Team

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SCIENCE

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Study guide

This Unit has three teaching components in addition to the Main Text: an Audio-vision sequence entitled 'Basic properties of light' (AC 91), a series of Home Experiments and a TV programme (TV 09). The Audio-vision sequence can essentially be regarded as Section 2 of the Main Text, and we recommend that you study it immediately after the Introduction. The tape lasts about 25 minutes and there are 35 frames on five filmstrips (9.1–9.5) which you should look at while listening to the tape.

The Home Experiments have been integrated with the Main Text, and we think that you will get most benefit if you can do them when you reach the appropriate sections of the text. Since all four experiments use the same light source, you will save yourself time if you can leave the light source set up on a bookshelf or on some other surface which is not disturbed. Home Experiments 1, 3 and 4 should only take about 10 minutes each; Home Experiment 2 is longer and may take about half an hour, including working out the results.

The TV programme shows a number of experiments and demonstrations that should help you to visualize and understand the models of light that are discussed in the Main Text. It is not essential to have read the Main Text before watching this TV programme.

1 Introduction

Light is an important energy source that was only briefly mentioned in the wide-ranging discussion of energy in the previous Unit. It provides the energy for photosynthesis, without which the plants that we know today would not exist (and so neither would we!), and without which the Earth's stocks of fossil fuels would not have been built up in past millennia. It will become even more important to our everyday existence when our stocks of fossil fuels are exhausted in the not-too-distant future. Mankind will probably have to depend on solar power generation (that is, electricity generated directly or indirectly from sunlight) to meet its needs for more and more joules of energy. However the importance of light is due not only to its energy-carrying properties, but also to the fact that it varies from one place to another, and from one instant of time to another, and these variations convey a vast amount of information. There is no doubt that the major part of the information that we receive about the world around us is carried by light. Indeed the information that you are receiving at this moment from this printed page is being carried by light.

What is light? That is a question that has puzzled people from the beginnings of history. What is it that enters our eyes to produce the sensation of vision? What is it that carries energy across the vast distance between the Sun and Earth? We would like some kind of model that would help us to understand how light is produced, how it travels from one place to another, how it interacts with various objects, how it is absorbed, and so on.

In the earlier Units we discussed two quite distinct ways in which energy can be transported from one place to another. One of these ways is wave motion, of which seismic waves are an important example. The other is particle motion; that is by the movement of particles, each with a certain amount of kinetic energy, from one place to another. Each of these means of energy transport forms the basis for a possible model of the behaviour of light.

In this Unit we shall compare two models of light—a wave model and a particle model—and consider which of various properties of light are best explained by the one model or by the other. We shall also investigate the following question: if light is to be represented by waves or by particles, what sort of waves must these be, or what sort of particles? Our aim is to end up with a model that can explain all facets of the behaviour of light.

2 Modelling basic properties of light

We shall start by discussing six properties of light, each of which is part of everyday experience. In each case we shall ask whether wave and/or particle models can account for the property.

Study comment Since this discussion involves a large number of photographs and diagrams, we are presenting it as an Audio-vision sequence. During this sequence, we shall give many details to justify the use of wave and particle models, but you are not expected to memorize them all. You should aim chiefly to understand the extent to which the models explain various properties of light. To help you, there is a summary of the main points in Table 1. Although this Table will not be referred to until the end of the tape, you may like to have it in front of you while studying this sequence. Space has been left in the Table for you to make odd notes, if you find that doing this is helpful.

The Audio-vision sequence is called 'Basic properties of light'. It is recorded on audio cassette AC 91. It uses five strips of film and these have the reference numbers 9.1–9.5. *We recommend that you study this Audio-vision sequence before continuing with Section 3 of the Main Text.*

Filmstrips 9.1–9.5

2.1 Objectives of Section 2

Having studied this Audio-vision sequence, you should be able to explain how, and to what extent, the six main properties of light discussed in this sequence (namely straight-line propagation, decrease of intensity with distance, crossing

TABLE 1

Observed property of light	Wave model	Particle model
Travels in straight lines. Small source produces sharp shadows.	Ambiguous. Waves sometimes produce sharp shadows (seismic waves) but sometimes are diffracted round corners (water, sound).	Paint spray models this behaviour very well.
Intensity falls off with distance from source.	Water waves show this behaviour.	Paint spray model is good.
Crossed light beams do not scatter each other.	Ripples on water surface pass through each other without distortion.	Beams of particles scatter each other (unless particles are very small and widely spaced).
Reflection at smooth surface; angle of incidence i = angle of reflection R.	Water waves are reflected from smooth surface with $i = R$.	Billiard balls are reflected from smooth surface with $i = R$.
Reflection at rough surface; light reflected at all angles.	Waves reflected from rough surface at all angles.	Billiard balls reflected from rough surface at all angles
Light refracted on passing from one transparent material to another. $\frac{\sin i}{\sin r} = \text{constant}$ Larger angle in air than in water, i.e. $\frac{\sin(\text{angle in air})}{\sin(\text{angle in water})} = 1.33$ Definition: *refractive index* of a material $= \frac{\sin(\text{angle in air})}{\sin(\text{angle in material})}$ $= 1.33$ for water Note that this definition does not depend on which way the light travels.	Water waves change direction when they travel into region where velocity* is different. $\frac{\sin i}{\sin r} = \text{constant}$ $= \frac{\text{velocity before boundary}}{\text{velocity after boundary}}$ Larger angle in material of *higher* velocity	Particles change direction when they travel into region where velocity* is different. $\frac{\sin i}{\sin r} = \text{constant}$ $= \frac{\text{velocity after boundary}}{\text{velocity before boundary}}$ Larger angle in material of *lower* velocity.
	Contradiction: to be resolved by experiment	
Light shows partial reflection, as well as refraction, when passing from one transparent material to another.	Water waves and seismic waves show this behaviour.	Difficult to explain with a simple particle model.
Light can show total internal reflection.	Water waves and seismic waves show this behaviour.	Simple particle model can explain this.
Cut glass, water drops, etc. disperse light into rainbow of colours.	Dispersion of waves occurs if different 'colour' waves have different velocities.	Dispersion of particles occurs if different 'colour' particles have different velocities.

* When we use the term 'velocity', here and elsewhere in this Unit, we strictly mean the 'magnitude of the velocity', or the 'speed'.

of light from different sources, reflection, refraction, dispersion) can be accounted for by a wave model and/or by a particle model.

To test your achievements of these Objectives, try the following SAQs.

SAQ 1 Which property, or properties, of light listed in Table 1 can be:
(i) modelled by particles *and* by waves;
(ii) modelled by *neither* particles nor waves;
(iii) modelled apparently by some types of wave but not by others?

Answers to SAQs start on p. 48.

SAQ 2 Complete the following sentence using information from Table 1. Wave and particle models make .... predictions for the ratio of the .... of light in two different materials. These predictions are based on experimental observations of the .... of light and of waves and particles.

3 The velocity of light

We do not seem to have gone very far towards explaining whether light is particles or waves. With a few minor exceptions, all of the properties of light studied in the previous Section could be explained by either model. Maybe you think that this indicates that it does not matter which model is used. If both explained *all* phenomena (rather than the small selection discussed so far) equally well, then that viewpoint would be quite justified. However, in the remainder of this Unit you will see that there are some properties of light that cannot be explained by the particle model and some that cannot be explained by the wave model.

First of all, let us deal with the distinct difference in the predictions that the two models make for the ratio of the velocity of light in air to its velocity in water. It was pointed out in the Audio-vision sequence that when light travels from air into water it is refracted towards the normal to the interface; the angle of refraction r is less than the angle of incidence i.

According to the particle model presented in the Audio-vision sequence, is the velocity of light in water greater or smaller than its velocity in air?

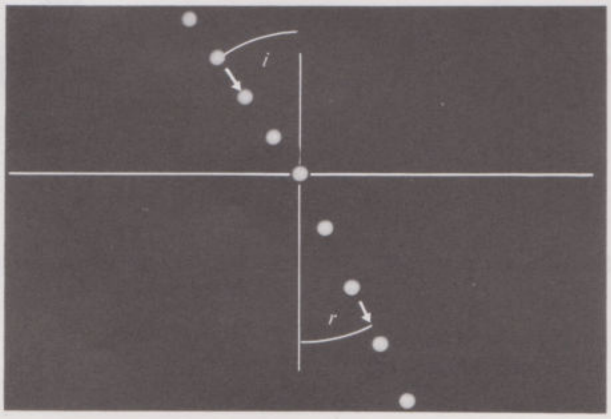
The velocity is *greater* in water. Remember that we had to speed up the particles by letting them go down a small ramp in order to get refraction towards the normal. We can think of the particles being speeded up by attractive forces very close to the interface.

What predictions does the wave model make about the velocity of light in air and water?

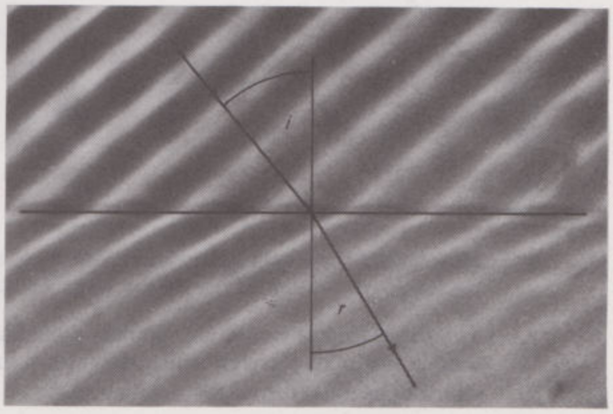
Exactly the opposite. As we pointed out in Unit 4, and also in the Audio-vision sequence associated with this Unit, waves obey Fermat's principle of least time. This predicts that refraction towards the normal will only occur if the velocity in water is *lower* than in air.

This difference is summarized in Figure 1, which appeared on the filmstrip in the Audio-vision sequence. With such a clear distinction between the predictions

FIGURE 1 The refraction of light as it travels from air to water can be modelled both by particles and by waves. However, the two models make different predictions for the ratio of the velocity of light in the two materials.



(a) $\dfrac{\sin i}{\sin r} = 1.33 = \dfrac{v_{\text{water}}}{v_{\text{air}}}$



(b) $\dfrac{\sin i}{\sin r} = 1.33 = \dfrac{v_{\text{air}}}{v_{\text{water}}}$

made by the two models, a decision on which is correct can be made if the velocity of light is measured in both air and water.

3.1 Measuring the velocity of light

Light travels at an extremely high velocity. For example, in air its velocity is $3 \times 10^8\,\mathrm{m\,s^{-1}}$, which is almost a million times faster than the Concorde supersonic airliner. This means that it takes very short times for light to travel any distance in a laboratory, and elaborate techniques are needed to determine these very short times if the velocity is to be measured.

A typical laboratory experiment to measure the velocity of light involves measuring the time required for light to travel a 10 m path length. About how long is this time?

3.3×10^{-8} s if the light travels through air. Since velocity is equal to (distance/time), it follows that:

$$\begin{aligned} \text{time required} &= \text{distance/velocity} \\ &= (10\,\mathrm{m})/(3 \times 10^8\,\mathrm{m\,s^{-1}}). \end{aligned}$$

Not very easy to measure!

The basic ideas behind a variety of methods which have been used to measure the velocity of light can be illustrated by considering the experimental arrangement shown in Figure 2a. In this experiment light from the small source S strikes a mirror M and is reflected to another mirror N. It strikes this mirror at right angles and is therefore reflected back to mirror M and, from there, travels straight back to the source S. Now mirror M can be rotated at a constant speed in the direction indicated by the arrows in Figure 2a. As it rotates, light from the source will only be reflected towards mirror N when the rotating mirror is in the position shown in Figure 2a. However, the light reflected back from mirror N will return to find that mirror M has rotated to a different position, which is shown in red in Figure 2b. The light will therefore *not* be reflected back to S, but instead will be reflected along the red line towards S_1. The faster the light travels, the shorter the time it takes to travel from M to N and back, the smaller the angle of rotation of mirror M in that time, and therefore the smaller the displacement from S to S_1. In fact d_1, the distance between S and S_1, is inversely proportional to the velocity of light in air, c_{air}, that is:

$$d_1 \propto 1/c_{air} \qquad (1)$$

Note that the letter c is conventionally used to represent the velocity of light.

velocity of light, c

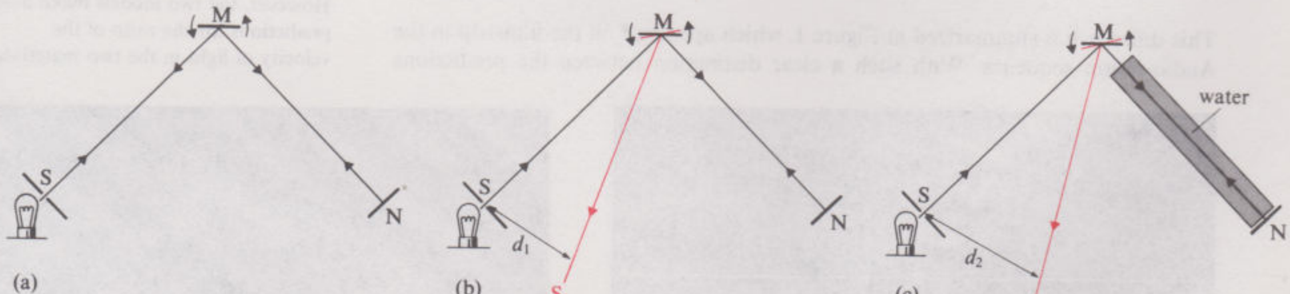


FIGURE 2 The rotating-mirror method for measuring the velocity of light.

How would the displacement from S to S_1 of the reflected beam change if the speed of rotation of the mirror were doubled?

The displacement would also double. Doubling the rotation speed would double the angle through which the mirror rotates in the interval during which light travels from M to N and back.

To calculate the velocity of light it is necessary to measure the speed at which the mirror rotates—so many revolutions per second—and the distances from M to N,

from M to S, and from S to S_1*. However, to compare the velocity in water with that in air we only need to insert a tube containing water into the apparatus, as shown in Figure 2c, and compare the new displacement d_2 with the displacement d_1 before the water was inserted. As long as the speeds of rotation, and the other distances involved, are the same for the two experiments, their actual values will not affect the ratio of the distances d_1 and d_2.

The result of this experiment is that the reflected beam is displaced through a distance that is 1.33 times *larger* when the light travels through water than when it travels through air, that is, $d_2 = 1.33\, d_1$. Now as shown in equation 1, beam displacement is inversely proportional to velocity, so:

$$\frac{d_2}{d_1} = \frac{1/c_{\text{water}}}{1/c_{\text{air}}} = \frac{c_{\text{air}}}{c_{\text{water}}} = 1.33$$

This result is just what is predicted if the experimental results on refraction of light are interpreted using a wave theory (see Figure 1b). It appears that we can decisively reject the particle theory as its prediction for the ratio $c_{\text{air}}/c_{\text{water}}$ is definitely at variance with experimental measurements.

3.2 Objectives of Section 3

Having studied Section 3, you should be able to do the following:

(a) Contrast the predictions made by wave and particle models for the ratio of the velocity of light in two materials when the angles of incidence and refraction are known.

(b) Explain why measurements of the velocity of light in various materials cause us to prefer the wave model to the particle model.

(c) Describe the rotating-mirror method of measuring the velocity of light, and perform calculations based on results obtained with this method.

To test your achievement of these Objectives, try the following SAQs.

SAQ 3 Experimental observations show that when light travels from a certain oil into water the angle of incidence in the oil is greater than the angle of refraction in the water.

(a) In which liquid does the wave model predict that the velocity of light is higher?

(b) In which liquid does the particle model predict that the velocity is higher?

(c) In which liquid would the measured velocity of light be higher?

SAQ 4 The rotating-mirror method is used to compare the velocity of light in two types of glass. This is done by comparing beam displacements when the tube of water in Figure 2c is replaced in turn by long rods made from the two types of glass. The velocity in glass A is known to be $2.00 \times 10^8\,\text{m}\,\text{s}^{-1}$ and the displacement of the reflected beam is 10.3 mm. The displacement with glass B in the apparatus is 10.0 mm. What is the velocity of light in glass B?

4 A closer look at the wave model

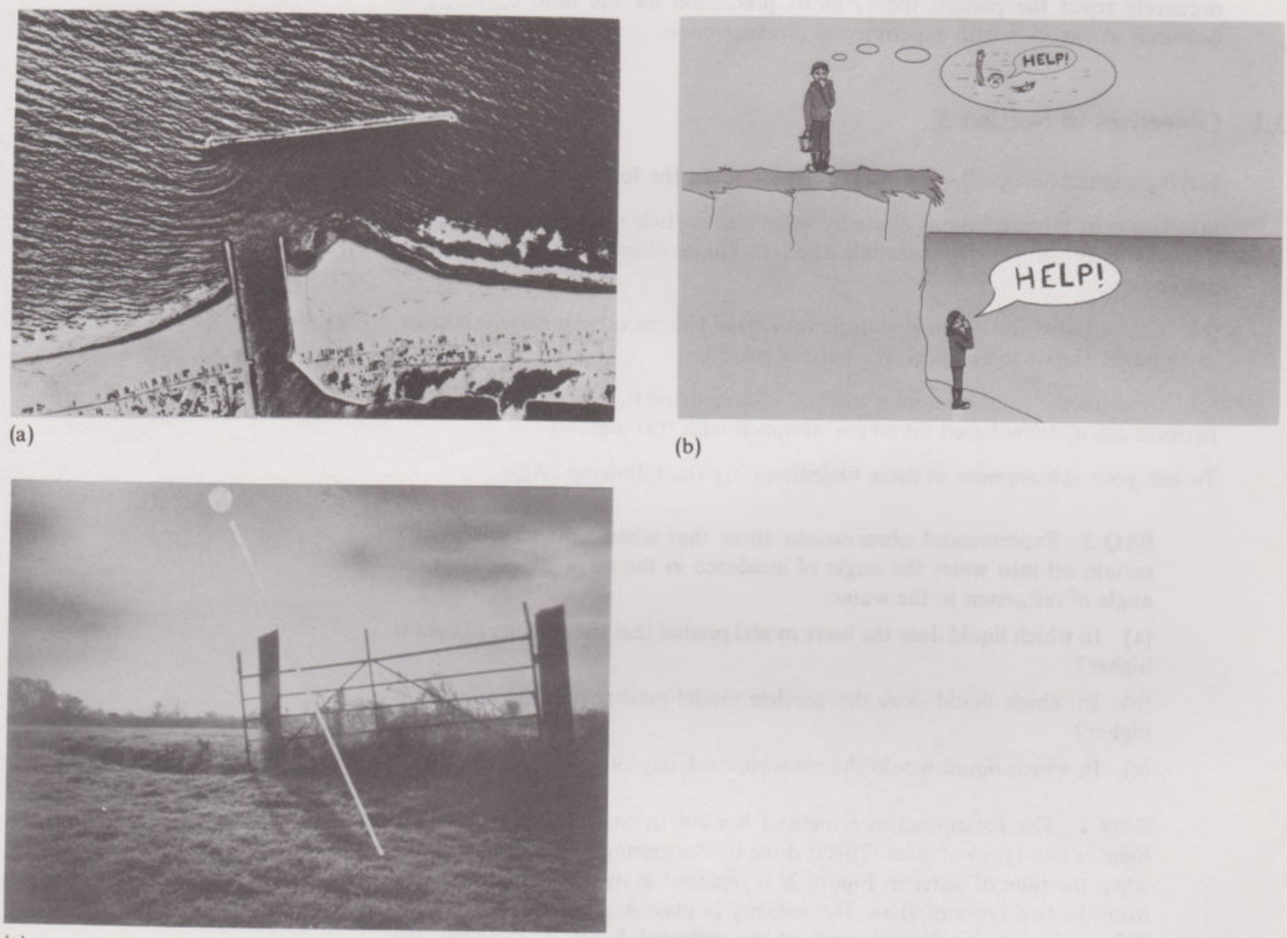
In Section 2 and the associated Audio-vision sequence we took various well-known properties of light and explained them in terms of both a wave model and a particle model. Since the particle model was subsequently rejected because it could not explain the velocity measurements, we can now concentrate on the wave model and see what other properties of light it can explain. However, we

* You may be wondering about the practical feasibility of this experiment. In fact if the distances from M to N, and from M to S, are 15 m, and if the mirror rotates at 200 revolutions per second, then the displacement from S to S_1 is typically about 4 mm.

shall take the opposite approach to the one used in Section 2: rather than taking a property of light and explaining it in terms of waves, we shall take a property of waves and ask if light behaves in a similar way. In particular, we shall confront a problem still outstanding from earlier in the Unit: why are water waves and sound waves diffracted around corners, while light generally appears to travel in straight lines?

4.1 Diffraction of waves

The apparent difference between light and sound and water waves is clearly shown in Figure 3, which you should recognize from the Audio-vision sequence. Light casts sharp shadows; sound and water waves are *diffracted*, that is they can travel around corners. It is not surprising that this contrasting behaviour was at one time used as evidence *against* the wave model of light. **diffraction**



(a) (b) (c)

FIGURE 3 There appear to be significant differences between the ways that waves travel: (a) water waves, (b) sound waves, and (c) light waves.

The example of water wave diffraction shown in Figure 3a is rather complicated, and in order to make progress in resolving the differences between the behaviour of light and water waves it is helpful to look at a simpler example of the diffraction of water waves. Such an example is shown in Figure 4a. In this photograph you can see parallel ripples on the surface of water striking a barrier in which two slits have been cut. The waves that get through the slits spread out—they are diffracted—but they do not spread out equally in all directions. There is a set of waves that keeps going in the same direction as the original parallel waves, and there are four other clearly visible sets of waves. The most interesting—and significant—aspect of this picture is that the water surface midway between adjacent sets of waves is stationary. Somehow the waves that get through the two slits are cancelling each other in these intermediate regions. What is more, the positions of these regions of cancellation depend on the separation between the two

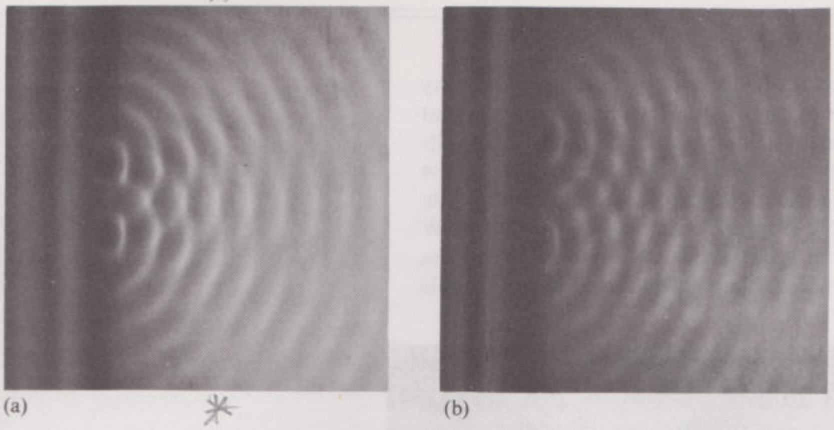


FIGURE 4 Water ripples are diffracted when they pass through a pair of slits. The ripples are moving from left to right across the picture. The crests of the ripples appear bright and the troughs appear dark in this photo because of the method of illumination. The distance between wave crests was 17 mm in both cases; in (a) the slit spacing was 40 mm and in (b) 60 mm.

slits, as you can see by comparing Figure 4a with Figure 4b. The smaller slit spacing in Figure 4a gives rise to a larger spacing between the regions where the water is stationary.

Before taking up the question of exactly how and where this cancellation occurs, we would like you to check that the same kind of effect occurs with light. After all, if the wave model is a good description of the behaviour of light, then there must be an analogous effect that can be observed when two slits are illuminated with light. If no such effect were observable, doubts would naturally arise about the correctness of the wave description of light.

4.2 Diffraction of light by two slits; Home Experiment 1

It is comparatively easy to set up an experiment with light that is analogous to the experiment with water ripples shown in Figure 4. The essential ingredients are:

1 parallel light waves that strike

2 a pair of slits, and

3 a detector to investigate the angles at which the light leaves the slits.

Waves that are essentially parallel can be obtained using a small, distant source of light. As we have indicated in Figure 5, the waves produced by a small source are really spherical. However, if we are only concerned with a small region that is a long way from the source then the curvature of the waves can be neglected.

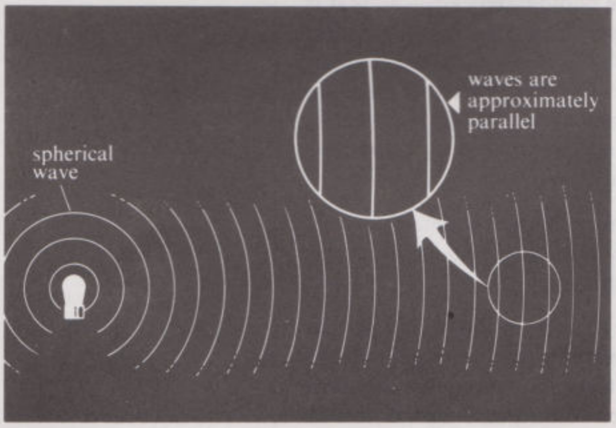


FIGURE 5 The waves produced by a small light bulb. You can think of the lines as representing the crests of the light waves: they spread out in a similar fashion to the water ripples produced when a pebble is dropped into water.

Home Experiment 1

You can produce what is effectively a small source by putting an ordinary light bulb behind a couple of pieces of black card which have a gap of about 1 mm between them. This arrangement is shown in Figure 6. The light bulb should be inserted in the bulb-holder assembly that is provided in the Home Experiment Kit for use with the moth trap. You will find that you get the best results if you use a 100 W pearl bulb, though 60 W or 40 W bulbs will give reasonable results in a darkened room. Do **NOT** use the bulb provided in your Kit; it is a mercury vapour bulb, and is not suitable for the experiments in this Unit.

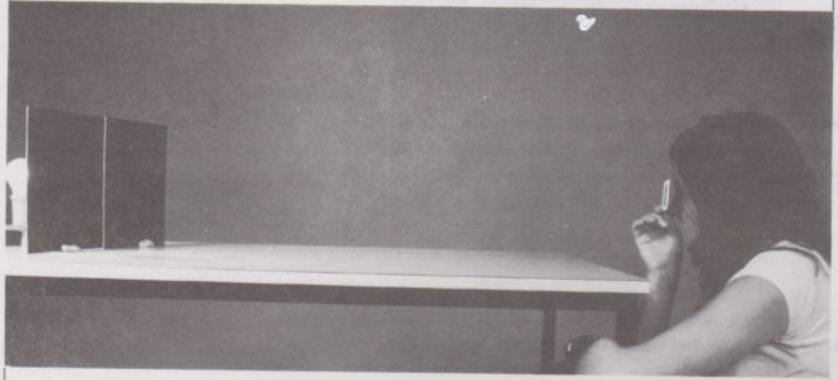


FIGURE 6 The arrangement suggested for observing the diffraction of light by the double slits. Note that the double slits should be parallel to the slit between the black cards.

The cable from the bulb holder must be plugged into the extension socket leading from the isolating transformer. You will need to fit a three-pin mains plug to the cable of the isolating transformer. If you are unsure about how to connect the plug, then you should refer to the wiring instructions for three-pin plugs in the booklet for the Home Experiment Kit, Part 1. The moth-trap assembly instructions in that booklet include a checklist that you should follow if the bulb does not light up.

You can support the two black cards with Blu-tack, and you should keep them at least an inch from the light bulb so that they do not get too hot. Since you need to use this same arrangement of light bulb and cards for a number of other experiments later in this Unit, and also in Units 10 and 11, it would be worthwhile setting it up in a position where it can be left undisturbed – maybe at the back of a desk or a sideboard, or on a bookshelf. Take care that the cables are not stretched, and that they are not left in a position where they can trip somebody or where a young child can pull the bulb holder down. The isolating transformer should not be covered over, or it may overheat.

A pair of light-transmitting slits can be made photographically—all that is needed is a piece of film which is opaque apart from a pair of transparent lines. We have provided you with three double slits in your Home Experiment Kit with spacings of 0.08 mm, 0.16 mm and 0.32 mm. All of them are on the same 35 mm transparency. Don't worry about the 'grating' that is on the same transparency, or about the *single* slit that is below the 0.080 mm double slit—they are used in later experiments.

The remaining important component in this experiment, the detector, is not included in the Home Experiment Kit. This is because you have suitable light detectors readily available, namely your eyes.

The way that you should do the experiment is shown in Figure 6. You hold the 35 mm transparency close to your eye, with the double slits parallel to the gap between the black cards. Look at the light source through each double slit in turn from a distance of about one metre, and make a note in Table 2 of what you see.

Try this experiment now, and then answer the following two questions before reading the comments in Appendix 1, on page 46.

TABLE 2 Observations of diffraction by double slits

Spacing of double slit	Observations
0.08 mm	
0.16 mm	
0.32 mm	

In what ways are your experimental observations similar to the results of the water wave experiment shown in Figure 4, and in what ways are they different?

What effect does the spacing of the slits have on what you observe?

4.3 Characterizing a wave

The basic similarity between the diffraction of water and light by double slits is certainly additional evidence for the wave-like nature of light. It is also additional evidence *against* a particle model, since the observed behaviour appears to be inexplicable in terms of our experience of the way in which billiard balls and paint droplets behave. For example, if you rolled billiard balls at a screen with two slits in it, then you would expect to observe a 'stream' of balls coming straight through each slit. In addition, some balls would collide with the edges of the slits and would be deflected through various angles. The balls would certainly not divide up into five or more clearly defined streams, and so this model cannot explain your experimental observations of light diffraction.

Having made the point that diffraction can be explained by waves but not by particles, we could conclude our discussion of this topic. However, we can learn a great deal more about light by delving deeper into the subject of diffraction, by investigating how it occurs, and what causes the mysterious dark fringes, and the regions of stationary water. Our long-term aim is to refine the wave model of light further, but first we need to establish a vocabulary for talking about waves, and for discussing what quantities characterize their behaviour. To do this we will go back to water waves, or ripples, since these are the easiest type of wave to visualize.

A water wave is simply a variation in the surface level that travels across the surface of the water. The waves that you observe at the seashore are generally very complicated, but much simpler waves can be produced in a laboratory tank. Let us consider, for example, the simple parallel waves shown in Figure 7a. The height of the water surface along the line AB in that figure can be plotted on a graph, and the result of doing this is shown in Figure 7b. The graph simply represents a vertical cross-section through the water in the direction AB, and clearly shows the *crests* and *troughs* of the waves (the highest and lowest points on the surface).

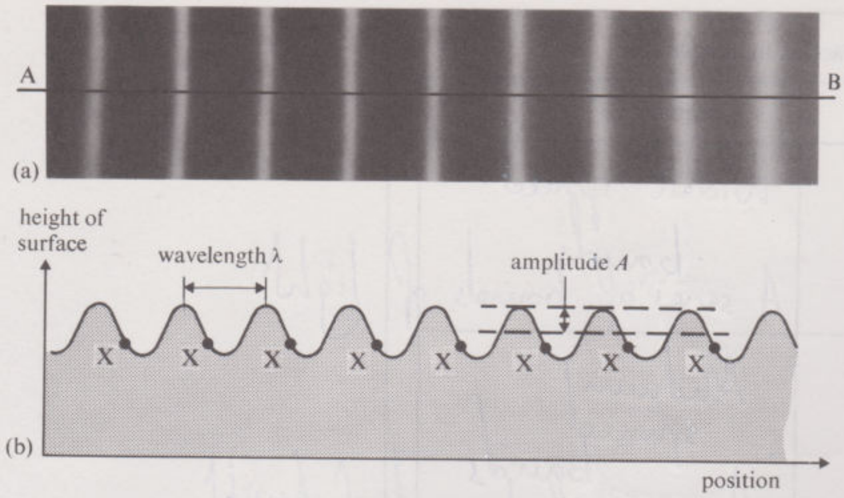


FIGURE 7 Water waves produced in a laboratory tank. (a) A photograph of parallel ripples on the surface of water. (b) A graph that represents a vertical cross-section through the water surface along the direction AB.

This cross-sectional picture is ideal for defining two of the important parameters, that is, two of the important variable quantities, that characterize a regular wave such as this. As you have probably noticed in Figure 7, the distances between successive crests are identical, and they are equal to the distances between successive troughs. This same distance separates successive points marked with an X, and also any other set of points at equivalent positions on the surface. This characteristic distance is called the *wavelength*, and it is generally denoted by the small Greek letter λ, pronounced *lamb-da*. In Figure 8 you can see the cross-sectional profiles of a number of waves that have different wavelengths.

wavelength, λ

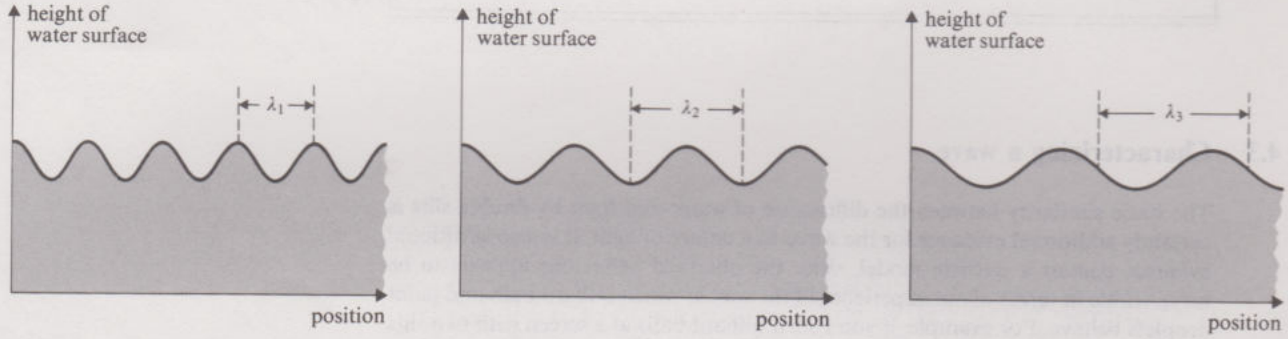


FIGURE 8 These graphs represent water waves with three different values of *wavelength*.

The other simple parameter describing the wave in Figure 7 is related to the difference in height between the crests and the troughs. In fact it is conventional to describe the height variation by stating the difference between the average level of the wave and its highest (or lowest) points. This difference is called the *amplitude* of the wave, generally denoted by A. In Figure 9 you can see the profiles of a number of waves that have different amplitudes, but identical wavelengths.

amplitude, A

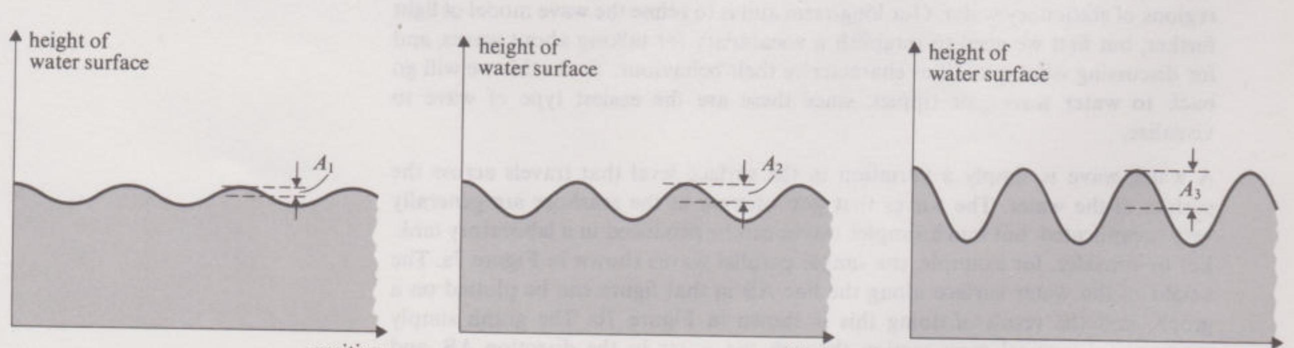


FIGURE 9 These graphs represent water waves with three different values of *amplitude*.

The amplitude is important because it is related to the energy carried by a wave. If you have watched waves rolling onto the seashore, particularly on a stormy day, you will have realized that the greater the height or amplitude of the waves, the greater is the energy they carry with them. In fact, the energy carried is proportional to the square of the amplitude,

$$E \propto A^2$$

so doubling the amplitude of a wave leads to a fourfold increase in the amount of energy that it carries. It is often more convenient to consider the amount of energy that the wave carries in one second across a line one metre long perpendicular to the direction in which the wave is travelling; this is called the *intensity* of the wave, and we will represent it by the symbol I*. From this definition, the intensity is proportional to energy carried, and since energy carried is proportional to the square of the amplitude, the intensity must also be proportional to the square of the amplitude, that is:

intensity, I

$$I \propto A^2$$

The photographs and graphs that have been presented so far have all shown waves at a single instant in time, and they are therefore limited in their ability to represent the *motion* of waves. We can, of course, take a series of photos, and draw a series of graphs that represent the form of the wave at successive instants of time; this has been done in Figure 10. In the short time between taking photos (a) and (b), the waves have moved about one third of a wavelength ($\lambda/3$) to the right. The same shift of the position of the crests is apparent if graph (b) is compared with graph (a). In part (c) of Figure 10 the waves have advanced $2\lambda/3$ beyond the position shown in (a), and in (d) the waves have advanced by a distance $3\lambda/3$, that is, λ. The most significant fact about these pictures is that (d) is exactly the same as (a). When the wave has travelled through a distance equal to the wavelength, the water surface looks exactly the same again.

The time that the wave takes to advance by one wavelength, that is, the time interval between two instants at which the wave profile looks the same, is another important parameter that characterizes a wave. This time interval is called the *period* of the wave, and it is generally denoted by the letter T.

period, T

> Bearing in mind the definition of the period of a wave (and also the definition of velocity), how is the velocity v of the wave related to T and λ?
>
> Velocity is the distance travelled divided by the time taken; the wave travels a distance λ in time T. Thus
>
> $$v = \lambda/T \qquad (2)$$

Note that the term period has exactly the same meaning here as it had in earlier Units. A period of one day is the time required for the Earth to rotate once about its axis. The period of one year (approximately) is the time taken for the Earth to travel around the Sun and return to its starting point. In each case, after an interval of one period, the system appears the same again: the water surface looks exactly the same; the same line of longitude faces the Sun; the Earth is in the same position in its orbit.

Closely related to the period of a wave (or any other periodic phenomenon) is its *frequency*. This parameter is essentially the answer to the question 'How frequently?' How frequently does the earth travel around its orbit? With a frequency of one revolution per year, or one revolution in $(365 \times 24 \times 60 \times 60)$ seconds, or 3.2×10^{-8} revolutions per second. How frequently does a wave advance by one wavelength? Once per period, or with a frequency of (1/period) times per second, assuming that the period is expressed in seconds. So the frequency is simply the reciprocal of the period; we shall represent it by the letter f, and can therefore express the relationship between frequency and period by the simple equation:

frequency, f

$$f = 1/T \qquad (3)$$

* In the case of light, sound, and all other types of waves that travel in *three* dimensions, the intensity is the energy carried per second across an *area* one metre square perpendicular to the direction in which the wave is travelling.

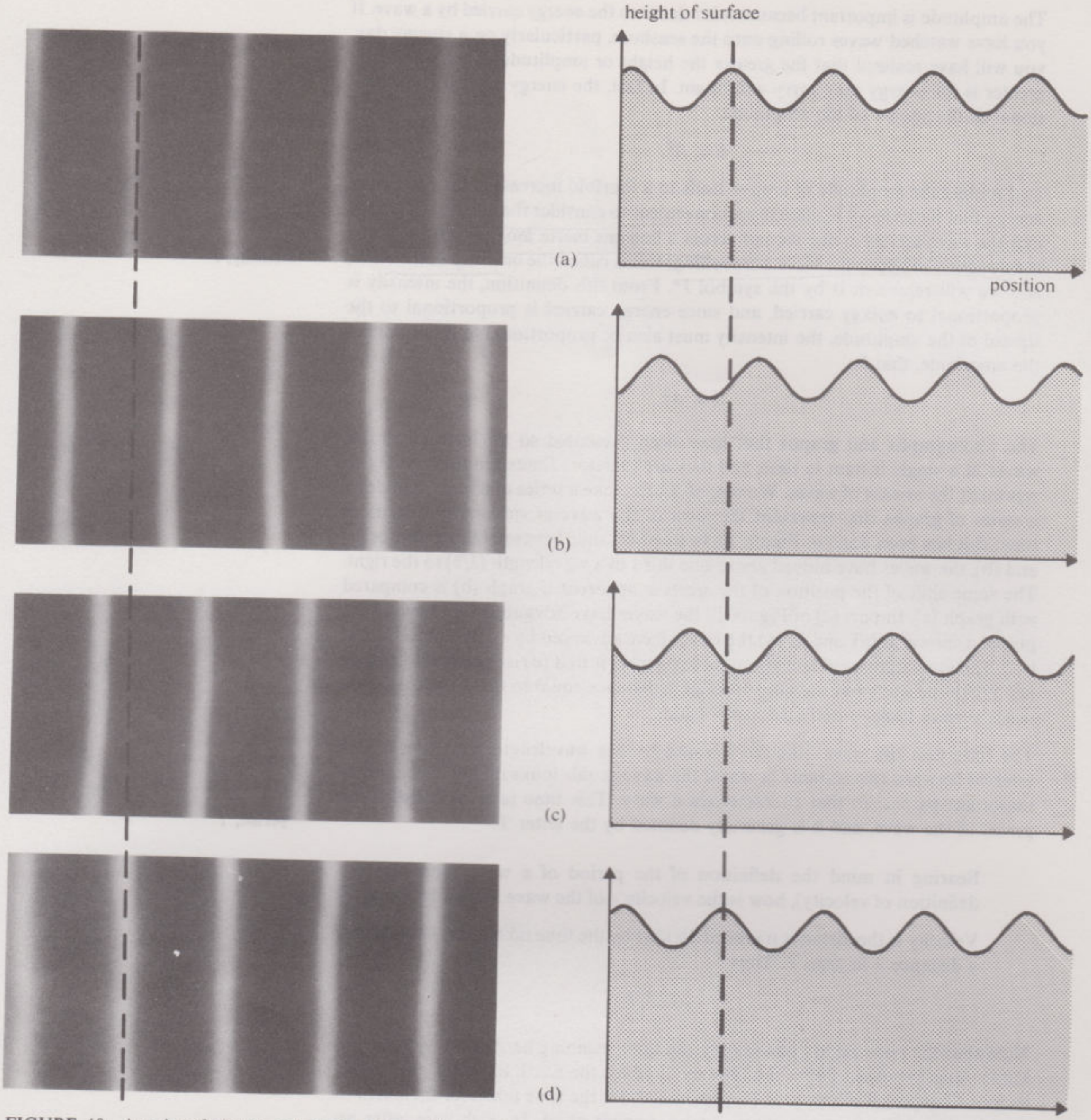


FIGURE 10 A series of photos and graphs representing a water wave at four successive instants of time. The wave has advanced by one third of the wavelength (that is, $\lambda/3$) between successive pictures.

Of course, we can combine this equation with equation 2 to produce an equation that relates the velocity of a wave to its wavelength and frequency:

$$v = \frac{\lambda}{T} = \frac{1}{T} \times \lambda$$

and
$$\frac{1}{T} = f$$

so
$$v = f\lambda \qquad (4)$$

The unit of frequency in the SI system of units has been given the name hertz*, and this is usually abbreviated to Hz. It has dimensions $(\text{time})^{-1}$. One hertz is simply one cycle per second, and you will find that many people still quote frequencies as x cycles per second rather than x hertz.

* Named after Heinrich Hertz, the first person to demonstrate the existence of radio waves at the end of the nineteenth century.

Thus a wave that has a period $T = 0.002\,\text{s}$ has a frequency $f = 1/(0.002\,\text{s}) = 500\,\text{s}^{-1} = 500\,\text{Hz}$; another wave that has a frequency $f = 4 \times 10^6\,\text{Hz}$, or 4 MHz, has a period $T = 1/(4 \times 10^6\,\text{Hz}) = 2.5 \times 10^{-7}\,\text{s}$.

The method of illustrating the behaviour of a water wave at successive times that was shown in Figure 10 is rather restricted. We should need many more pictures, taken at closer time intervals, to represent the wave motion completely. However, there is a simpler way of representing pictorially the behaviour of the wave at different times. We can concentrate on *one position* in the path of the wave and plot a graph showing how the height of the surface changes with time at that position. Let us take the position marked X in Figure 11a: at the instant (which we arbitrarily call $t = 0$) represented by this graph there is a crest of the wave at X.

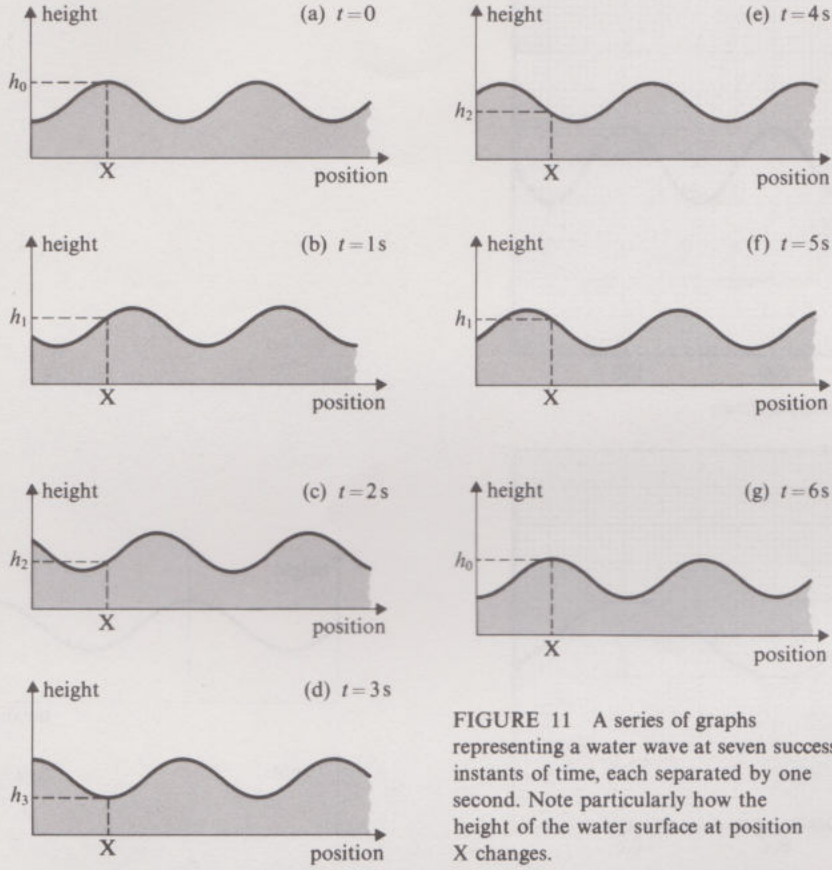


FIGURE 11 A series of graphs representing a water wave at seven successive instants of time, each separated by one second. Note particularly how the height of the water surface at position X changes.

The height of this crest is h_0 above some arbitrary reference level. One second later the wave has moved to the right, as shown in Figure 11b and the height of the surface at X has decreased to h_1. Figures 11c–11g show the waves at later intervals, and you can see that a trough (height $= h_3$) reaches X at $t = 3\,\text{s}$, and the next crest arrives at $t = 6\,\text{s}$. The information about the height at point X has been summarized in Figure 12, where the points labelled a, b, c and so on correspond to the times shown in Figures 11a, 11b, 11c, etc. If we had looked at the height at

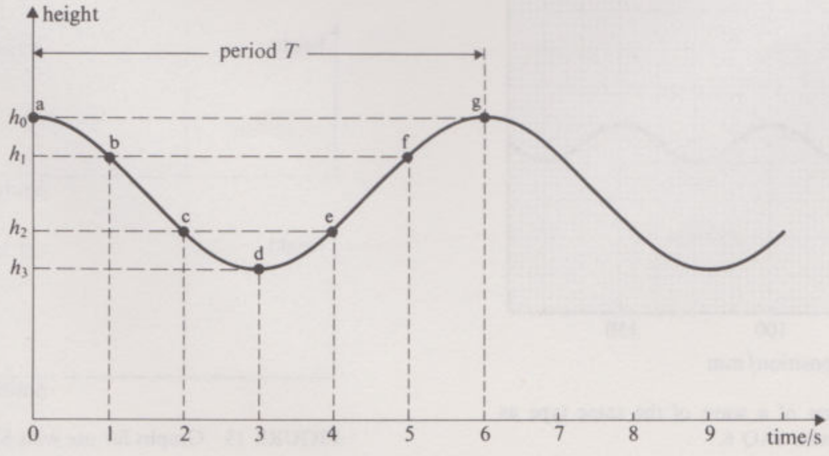


FIGURE 12 This graph summarizes how the height of the water surface at position X in Figure 11 changes with time. The points labelled a, b, c, and so on correspond to graphs (a), (b), (c), and so on in Figure 11.

X at any intervening instant of time we would have found that it could be plotted somewhere on the continuous curve that we have drawn through points a–g. Thus the curve in Figure 12 summarizes the effect that the wave has on the height of the water surface at position X. The period T of the wave is just the time taken for one cycle of height variation at X and this is obviously equal to 6 s.

4.3.1 Objectives of Section 4.3

Having studied Section 4.3, you should be able to do the following:

(a) Explain the meaning of the following parameters when used in connection with waves: wavelength, amplitude, intensity, period, frequency, velocity.

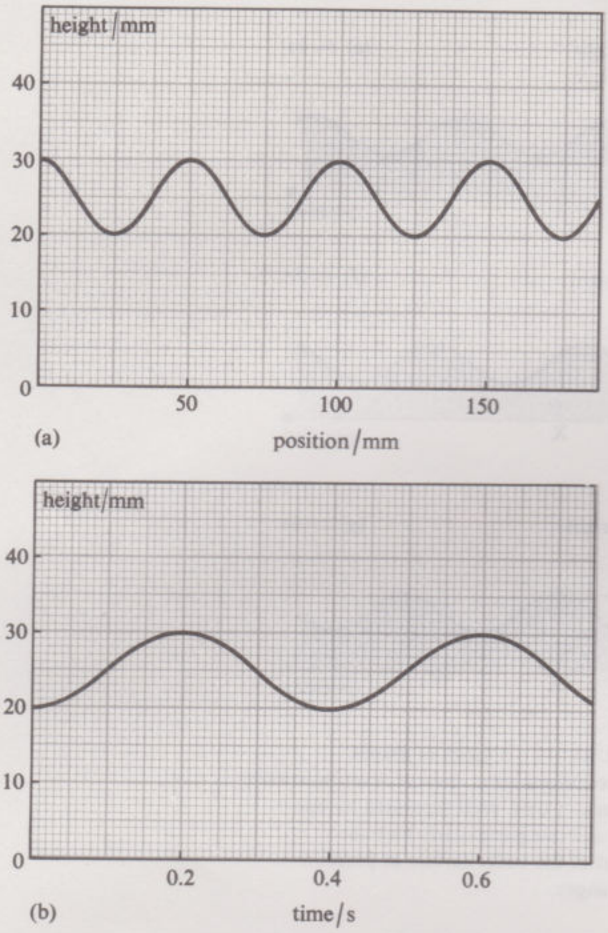


FIGURE 13 The alternative representations of a water wave. The graphs show the way that (a) the height of the surface depends on position *at a single instant in time*, and (b) the height of the surface depends on time *at a certain position*.

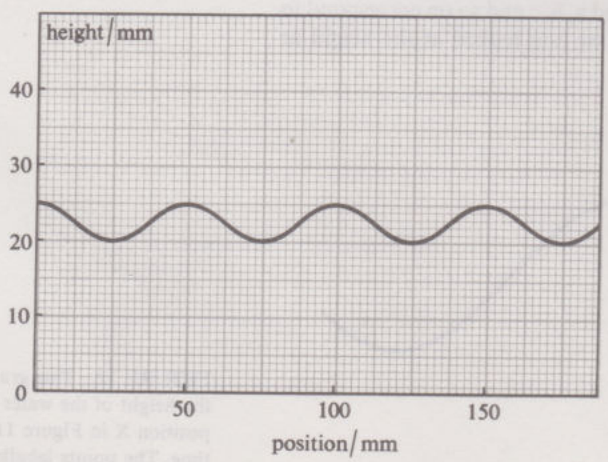


FIGURE 14 A representation of a wave of the same type as shown in Figure 13. For use with SAQ 6.

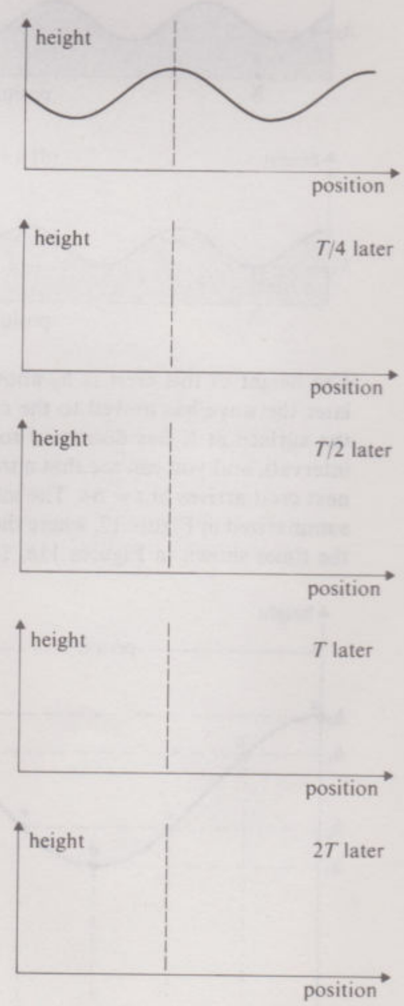


FIGURE 15 Graphs for use with SAQ 7.

(b) Recall that $v = f\lambda$, $f = 1/T$ and $I \propto A^2$, and use these relationships to calculate values of certain of these parameters when values of other parameters are specified.

(c) Relate these parameters to graphical representations of waves.

To test your achievements of these Objectives try the following SAQs.

SAQ 5 Two graphical representations of the same wave are shown in Figure 13. What are the wavelength, period, frequency, velocity and amplitude of this wave?

SAQ 6 Figure 14 represents a wave of the same type as that shown in Figure 13. What is the ratio of the intensity of the wave in Figure 13 to the intensity of the wave in Figure 14?

SAQ 7 The graph at the top of Figure 15 represents a water wave that is moving to the right. Sketch graphs below this that represent the same wave at instants of time that are $T/4$, $T/2$, T, $2T$ later, where T is the period of the wave.

4.4 Superposition of waves

Having dealt with the way in which a single wave can be characterized, we shall now ask what happens when two or more waves cross the same region. A complicated example of this is shown in Figure 16, where you can see the ripples spreading out from a number of raindrops. As was pointed out in the Audio-vision sequence, after passing through the region of overlap, each set of ripples continues on unaffected by the others—these ripples keep their characteristic circular shape. But what happens in the region of overlap?

The answer to this question is simple: we merely add the effects of the individual waves. To see how this works, look at the idealized example shown in Figure 17. Consider first the point labelled X, which lies in the region of overlap of two sets of water waves, both having the same wavelength and the same amplitude. The lines in this diagram represent the positions of crests of the waves. If only set A were present, then we could represent the height variation at X by the graph shown in Figure 18a. This graph is similar to that in Figure 12 (p. 17), except that we have plotted the height of the water surface above its *average* level. If only set B were present, then Figure 18b would represent the way that the height at X changed with time. What is the displacement caused by both sets together? We must simply add the two curves in (a) and (b). For example, at a time t_1, wave A on its own would raise the height of the water level by h_1, and at this instant wave B would also raise the height by h_1. So the effect of both waves together is to raise the level by $h_1 + h_1 = 2h_1$, and this is shown in Figure 18c. Since the effects of waves A and B separately are identical (that is, the graphs in Figure 18a and 18b are identical), the combined displacement at any time is twice as large as the displacement caused by one wave alone.

FIGURE 16 A photograph of the ripples produced by raindrops striking a water surface.

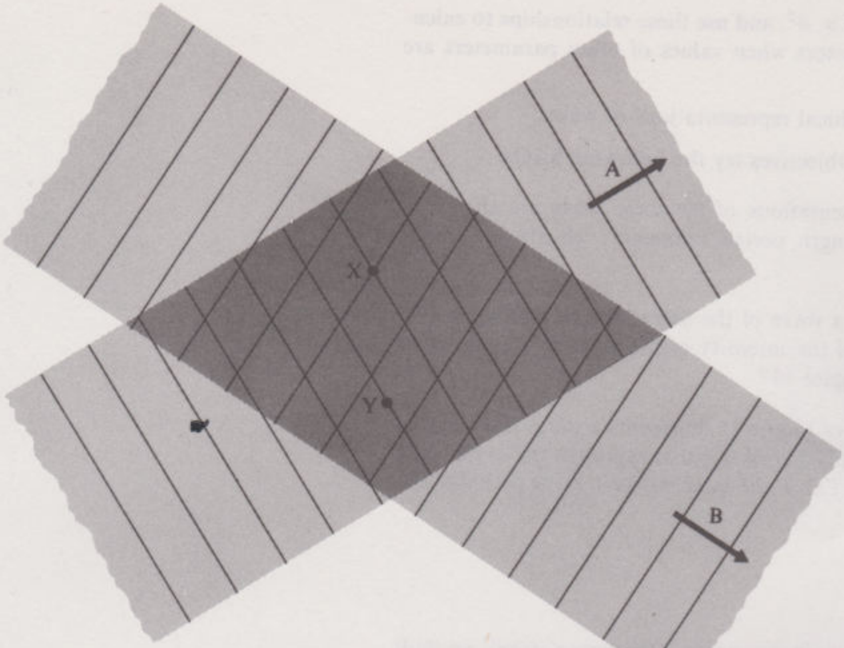


FIGURE 17 A representation of two sets of water waves crossing. The lines represent the positions of the crests of the waves in each set, assuming that the other set was not present.

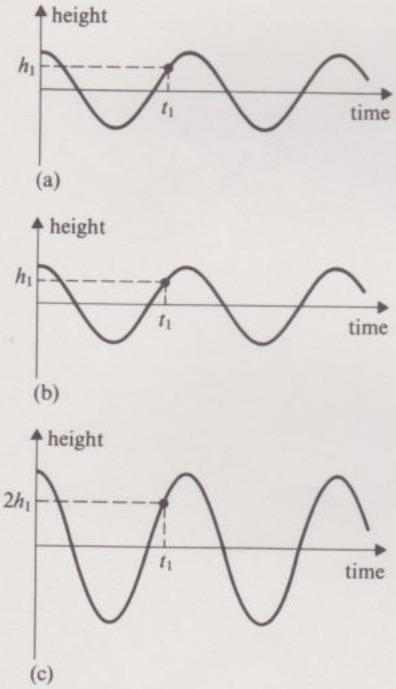


FIGURE 18 Displacement of the water surface shown at point X in Figure 17.

(a) The set of waves A would produce this displacement.

(b) The set of waves B would produce this displacement.

(c) The displacement at point X caused by both sets together is found by adding the graphs in (a) and (b).

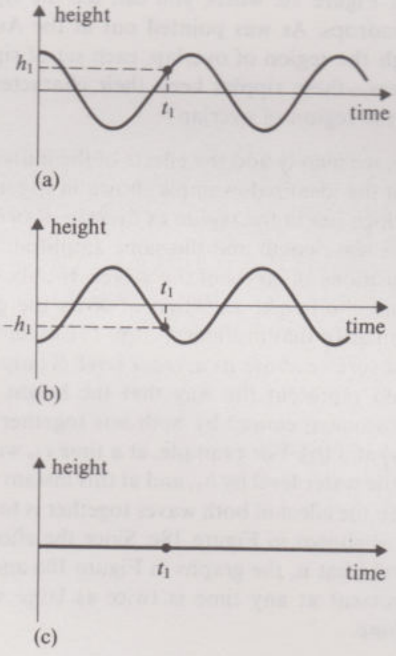


FIGURE 19 Displacement of the water surface shown at point Y in Figure 17.

(a) The set of waves A would produce this displacement.

(b) The set of waves B would produce this displacement.

(c) The displacement at point Y caused by both sets together is found by adding the graphs in (a) and (b).

What assumptions did we make that justify the fact that graphs (a) and (b) in Figure 18 are identical?

Because we wanted to start with the simplest situation, we assumed that waves A and B had the same wavelength (and the same period), that they had the same amplitude, and that crests of waves A and B reach point X simultaneously.

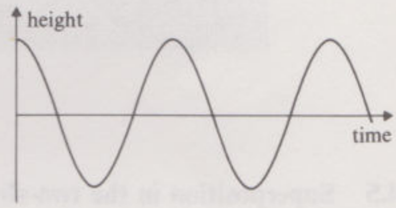
The way that the two sets of waves doubled up at point X may not seem too surprising, but look at the effect of the waves at point Y in Figure 17. At the instant shown in this diagram, a crest of wave A has reached Y, and a trough of wave B is passing through this point (since Y is midway between crests of wave B). One half period later a crest of wave B will reach Y, but the crest of wave A will have been replaced by a trough. In fact at any instant when a crest of one set of waves reaches Y, a trough of the other set of waves will get there too. This is illustrated in Figures 19a and 19b. Note the coincidence of the crests and troughs in Figures 19a and 19b, and note also that at any instant (such as t_1) the displacements of the two waves are equal in magnitude but opposite in sign. The combined displacement caused by the two waves at point Y is obtained by adding curves (a) and (b) in Figure 19, and this is shown in (c). *There is no movement at all of the surface at this point.*

At points X and Y in Figure 17 we see the extreme effects of these two waves overlapping. At other points the movement of the surface will be somewhere between these extremes, the amplitude being somewhere between zero and twice the amplitude of the individual waves. However, whatever position we consider, whatever instant of time we are interested in, and whatever the number of contributing waves, the principle used to find the combined displacement caused by all of the waves at a position is exactly the same: the total, or resultant, displacement is the sum of the displacements due to the individual waves. This principle is known as the *superposition principle*, and the superposition (or adding together) of waves is known as *interference*. At point X in Figure 17, where the two waves add to produce twice the amplitude of one of the individual waves, the interference is described as *constructive*. Conversely at point Y, where the two waves cancel out altogether, the interference is described as *destructive*.

superposition principle
interference

constructive interference
destructive interference

Though the superposition principle was introduced using a very simple example—two waves with identical amplitudes and identical wavelengths—it is of much more general validity. Figure 20a shows the superposition of two waves with different periods, Figure 20b shows the superposition of two waves with different amplitudes, and Figure 20c shows the superposition of three waves, each having a different amplitude and a different period. In each example the resultant wave (at the bottom of the column) is the sum of the component waves above it.



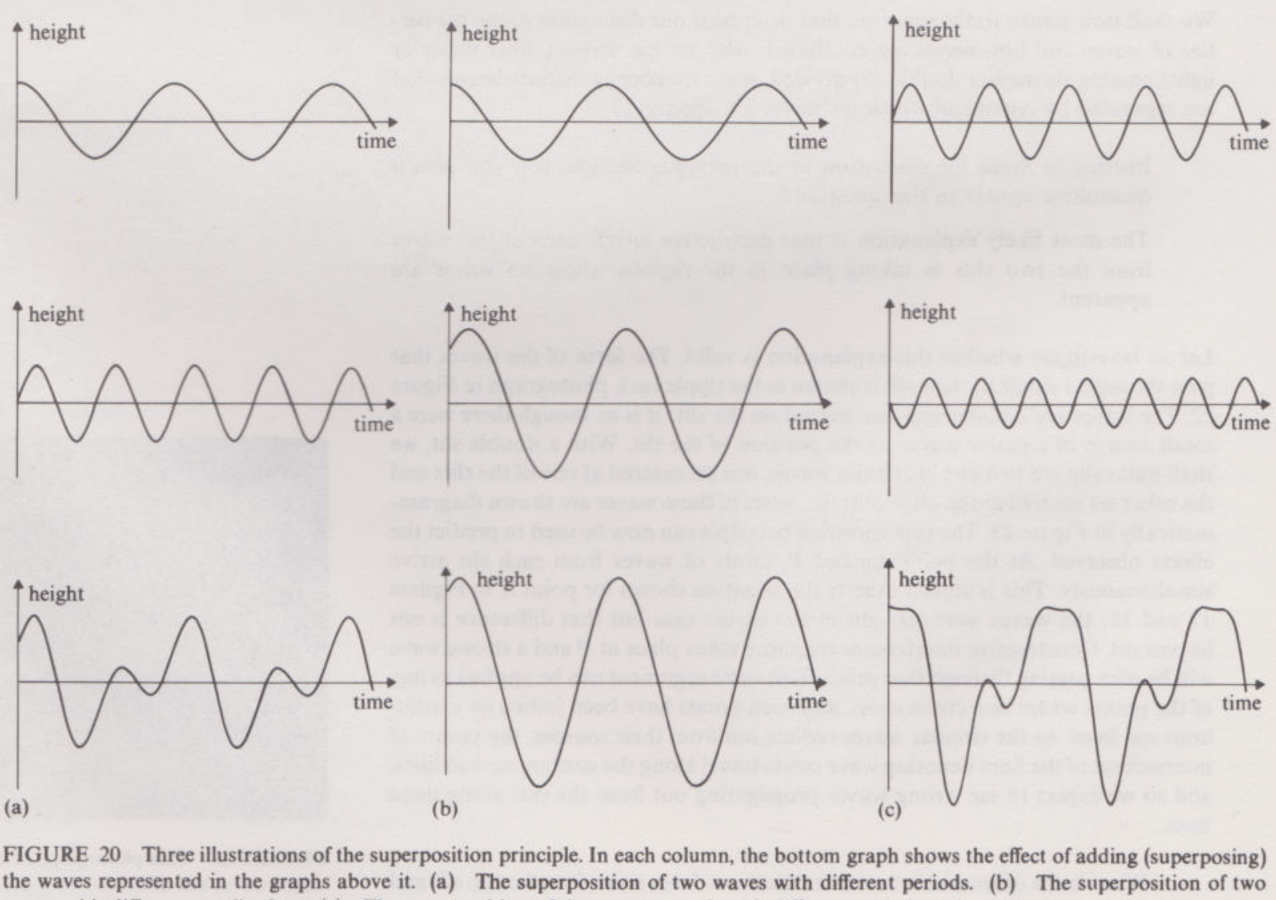


FIGURE 20 Three illustrations of the superposition principle. In each column, the bottom graph shows the effect of adding (superposing) the waves represented in the graphs above it. (a) The superposition of two waves with different periods. (b) The superposition of two waves with different amplitudes. (c) The superposition of three waves, each with different periods and amplitudes.

4.4.1 Objectives of Section 4.4

Having studied Section 4.4 you should be able to state the principle of superposition and apply this principle to simple problems.

To test your achievement of this Objective, try the following SAQ.

SAQ 8 The displacements produced at a certain point by two separate water waves are shown in Figure 21. Sketch on this figure the resulting displacement due to both waves together. (There is no need to be too precise about this.)

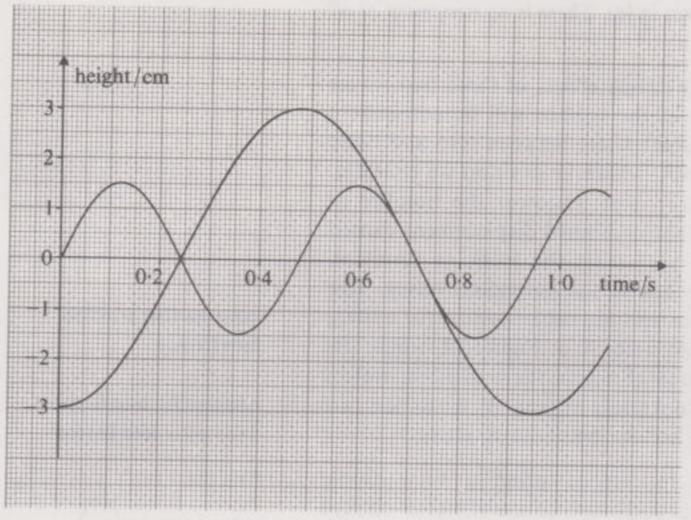


FIGURE 21 For use with SAQ 8.

4.5 Superposition in the two-slit problem

We shall now return to the question that prompted our discussion of the properties of waves and how waves are combined: why are the waves (either water or light) passing through a double slit divided into a number of distinct beams that are separated by regions in which no waves are apparent?

Bearing in mind the discussion in the previous Section, can you give a qualitative answer to this question?

The most likely explanation is that destructive interference of the waves from the two slits is taking place in the regions where no waves are apparent.

Let us investigate whether this explanation is valid. The form of the waves that pass through a single narrow slit is shown in the ripple tank photograph in Figure 22. The waves are circular and are centred on the slit: it is as though there were a small source of circular waves at the position of the slit. With a double slit, we shall naturally see two sets of circular waves, one set centred at one of the slits and the other set centred at the other slit; the crests of these waves are shown diagrammatically in Figure 23. The superposition principle can now be used to predict the effects observed. At the point marked P, crests of waves from each slit arrive simultaneously. This is almost exactly the situation shown for point X in Figures 17 and 18; the waves were straight in the earlier case but that difference is not important. Constructive interference therefore takes place at P and a strong wave will be seen passing through that point. This same argument can be applied to any of the points where two crests cross, and such points have been joined by continuous red lines. As the circular waves radiate out from their sources, the points of intersection of the lines denoting wave crests travel along the continuous red lines, and so we expect to see strong waves propagating out from the slits along these lines.

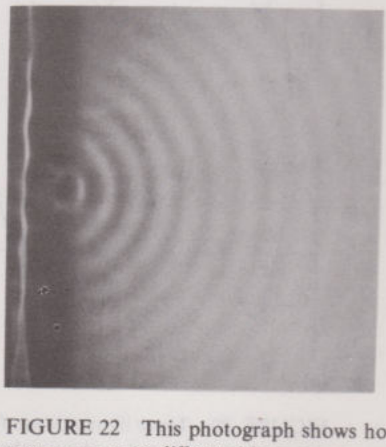


FIGURE 22 This photograph shows how water waves are diffracted by a narrow slit perpendicular to the water surface.

What is the difference between the distances from point P to the top slit and from point P to the bottom slit in terms of the wavelength?

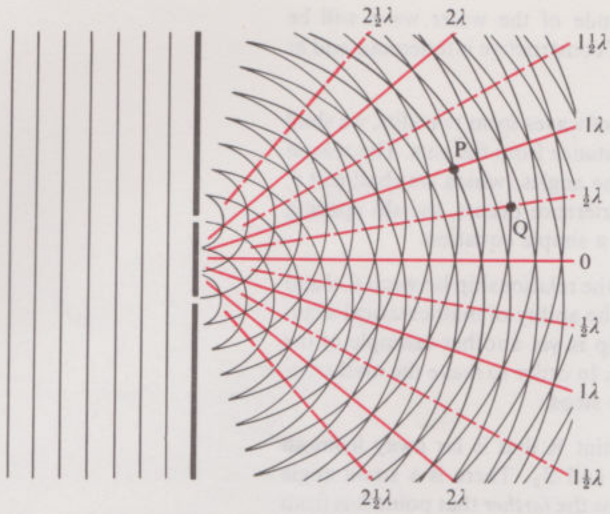


FIGURE 23 Diffraction by two slits perpendicular to this page. A set of circular waves is centred on each slit (as in Figure 22), and the black lines represent the crests of these waves. Along each of the red lines, the difference in the distance to the two slits has the value indicated at the end of the line. The solid red lines indicate constructive interference, the broken red lines destructive interference.

One wavelength, as can easily be demonstrated by counting the number of crests of each set of waves between the slits and P. There are 9 crests of the waves from the top slit and 10 crests of the waves from the bottom slit.

If you repeat this exercise for *any* point along the red line through P you will get exactly the same answer—one wavelength difference—and this is the reason that the waves arrive in step and interfere constructively. This difference in distance to the slits of one wavelength has been indicated at the end of the red line through P. Other lines of constructive interference correspond to other values for the difference in distance to the slits, and for each line the difference in distance has been indicated.

The important factor to note is that in each case the difference is an *integral* number (that is, a *whole* number) of wavelengths and that this must be the case because constructive interference requires the waves to arrive in step: peak with peak, or trough with trough.

Now consider the point marked Q in Figure 23. This lies on a crest of a wave from the upper slit, and on a trough (that is, halfway between two crests) of the wave from the lower slit. It is in a very similar situation to that shown for point Y in Figures 17 and 19, and so destructive interference must occur at Q. There are many other points in Figure 23 that lie on a crest of one set of waves and a trough of the other set, and they all lie on the broken red lines. These are the directions of destructive interference, along which the waves cancel out and the water surface remains still. At any point on the broken red line through Q the difference between the distances to the two slits is $\lambda/2$, half a wavelength, and you can easily confirm this for yourself by counting the crests of the waves. This is the reason that the two sets of waves always arrive out of step and therefore interfere destructively. Other lines of destructive interference correspond to other values for the difference in distance to the two slits, and for each line the difference in distance has again been indicated. Note that in each case the difference is an integral number of wavelengths plus an extra half wavelength, and this is because the waves must arrive out of step at any point on one of these lines.

So far we have only discussed interference at points at which the difference in distance to the two slits is either an integral number of wavelengths, or an integral number of wavelengths plus an extra half wavelength. If the first condition holds, then constructive interference occurs, and the amplitude of the resulting wave is twice the amplitude produced by one slit. If the second condition holds, then destructive interference occurs and there is no displacement of the water surface. Naturally, at most points neither of these conditions will hold. An intermediate

type of interference will occur, and the amplitude of the water wave will be somewhere between that observed at positions of constructive interference and at positions of destructive interference.

As the final step in our analysis of superposition of waves from two slits, we shall look at the interference that occurs at a large distance from the slits. The reason for doing this is that it will allow us to relate the angles (which we shall call θ, *theta*) at which constructive (or destructive) interference occurs, the slit spacing (which we shall call d) and the wavelength λ by a simple equation.

We need to use mathematical arguments to find the relationship between θ, d and λ for constructive interference to occur. Indeed, the ability of mathematical arguments to come up with the required relationship is yet another example of the utility and importance of mathematics in science. In order to make the mathematics easier to follow we have divided it into four steps.

Step A Look at Figure 24, which shows a point R that is far away from an opaque screen in which there are two slits S_1 and S_2. There is a small angle between the lines joining the slits to point R. Now the *farther* that point R is from the two slits, the *smaller* will be the angle between these two lines. Indeed, if the distances l_1 and l_2 between R and the slits are very much greater (1 000 times greater, say) than the distance d between the slits, then the angle between the two lines is so small that we can regard them as being *parallel* for the purpose of this calculation. This is the situation shown in the magnified inset in Figure 24: the lines from S_1 and S_2 to R are drawn parallel, and so they are inclined at the *same angle* θ to lines drawn at 90 degrees to the screen containing the slits.

Step B The line from slit S_2 to point M in Figure 24 has been drawn so that it is *perpendicular* to the lines between slit S_1 and R, and between slit S_2 and R.

This means that point M and slit S_2 are the *same distance* from point R, and this in turn means that the difference between distances l_1 and l_2 is equal to the distance between slit S_1 and point M; we use the notation S_1M as a shorthand way of representing this distance.

Step C We have already shown earlier in this Section that it is the difference between the distances l_1 and l_2 that determines whether constructive or destructive interference occurs at R. The next step, therefore, is to relate the distance S_1M

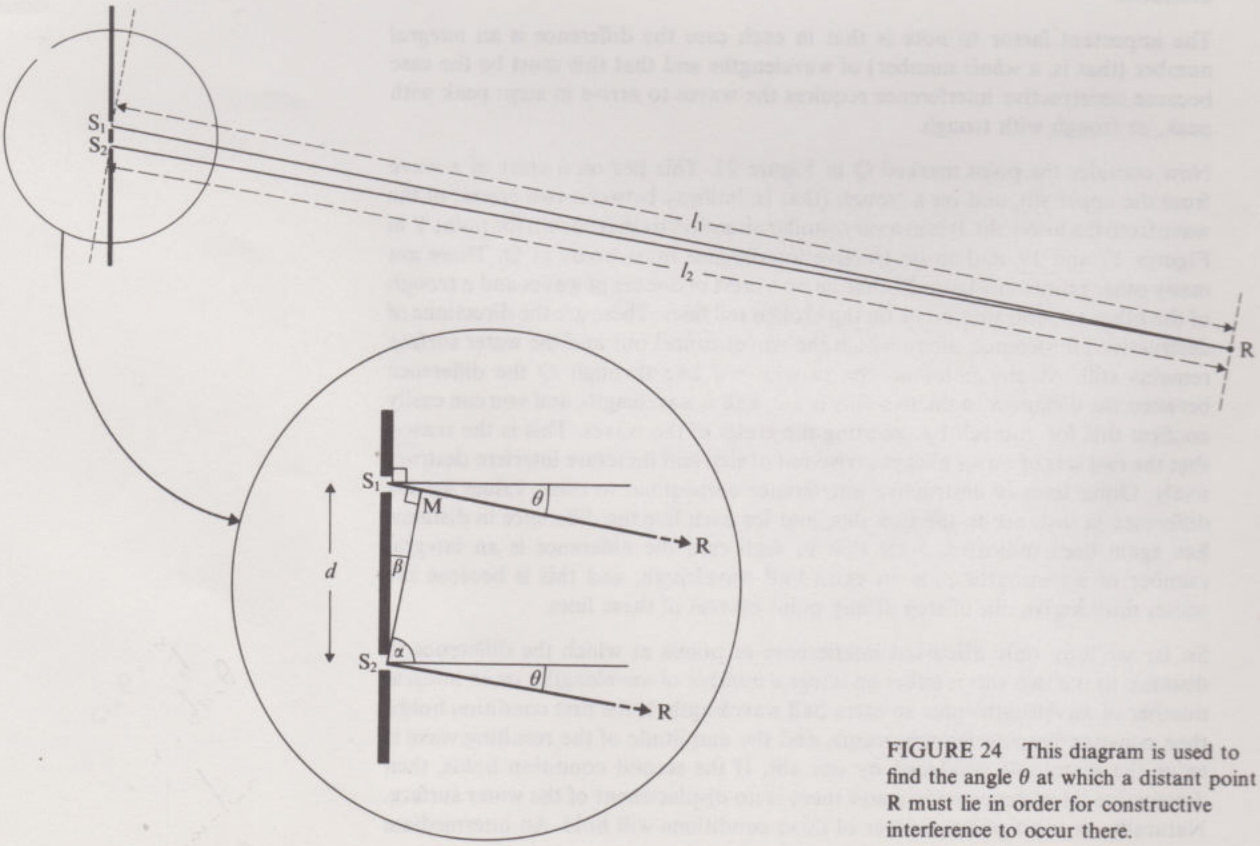


FIGURE 24 This diagram is used to find the angle θ at which a distant point R must lie in order for constructive interference to occur there.

to the angle θ and the slit spacing d. Because the line between S_2 and M was drawn perpendicular to the line between S_2 and R, we can say:

$$\text{angle } \theta + \text{angle } \alpha = 90 \text{ degrees}$$

But we also know that:

$$\text{angle } \beta + \text{angle } \alpha = 90 \text{ degrees}$$

since the sum of these angles is the angle between the screen and a line perpendicular to the screen. It follows from these two equations that:

$$\text{angle } \beta = \text{angle } \theta$$

Now, from the definition of the sine of an angle*

$$\sin \beta = S_1M/d$$

and so
$$\sin \theta = S_1M/d$$

or
$$S_1M = d \sin \theta \tag{5}$$

Step D For constructive interference to occur, the difference between the distances l_1 and l_2 must be an integral (i.e. whole) number of wavelengths, and so

$$l_1 - l_2 = S_1M = d \sin \theta = n\lambda$$

or
$$\sin \theta = n\lambda/d \tag{6}$$

where n is any integer. This equation tells us the angles at which constructive interference will be observed in terms of the wavelength λ and the spacing d of the slits.

The condition for *destructive interference* can readily be obtained by a slight modification of equation 6. You just need to remember that destructive interference requires a difference between the distances to the two slits of an integral number of wavelengths *plus* an odd half wavelength. The appropriate relationship for destructive interference is therefore:

$$\sin \theta = (n + \tfrac{1}{2})\lambda/d \tag{7}$$

Now look back to Figure 4 (p. 11) so that you can see how these quantitative equations tie up with actual observations of wave diffraction. First of all, as you may well have noted already, equation 6 tells us that the angles between the diffracted beams get *larger* when the slit spacing is made *smaller*. This is certainly apparent in Figure 4, and you should have observed the same effect in Home Experiment 1 (look back to the observations you recorded in Table 2 if you do not remember this.) However there is more information in equation 6 than the qualitative dependence of angle on slit spacing. In Figure 4a, the spacing d between the slits is about 2.4 times larger than the wavelength λ, that is, $\lambda/d = 1/2.4$. For constructive interference, equation 6 indicates that the values of θ should therefore be given by

$$\sin \theta = n/2.4$$

Thus

for $n = 0$, $\sin \theta = 0$ and so $\theta = 0$ degrees

for $n = 1$, $\sin \theta = 1/2.4 = 0.42$ and so $\theta = 25$ degrees

for $n = 2$, $\sin \theta = 2/2.4 = 0.84$, and so $\theta = 56$ degrees

A quick appraisal of Figure 4a should convince you that these are indeed the angles at which constructive interference occurs. Our mathematical equations seem to describe the experimental observations very well!

4.6 The wavelength of light; Home Experiment 2

Our wave model of light has been very qualitative until this stage in the Unit. It has seemed reasonable to describe light in terms of waves because of the *qualitative* similarities between the way that light behaves and the way that waves behave

* sine ≡ opposite/hypotenuse. You should refer to Block 4 of S101 *Mathematics for the Foundation Course in Science* (*MAFS*), The Open University Press (1978) if you are unhappy with this definition. MAFS 4

on the surface of water. The only parameter characterizing a light wave that has been discussed *quantitatively* is its velocity, which is $3 \times 10^8\,\mathrm{m\,s^{-1}}$ in air, and smaller by a factor of 1.33 in water.

However, having laboured through discussions of the characteristics of waves, having learnt how waves add up, and having produced an equation that describes the angles at which constructive interference occurs in the two-slit experiment, you are now getting to the pay-off for these efforts. By measuring the angles at which constructive interference occurs when a double slit (of known spacing) is illuminated with light, *you* can actually determine the magnitude of the wavelength of light. And, knowing the wavelength and velocity, you can calculate the period or frequency by using the equations $v = \lambda/T$ or $v = f\lambda$.

Home Experiment 2

For this experiment you need to make only a slight modification to the arrangement of light source and slits that you used for the earlier experiment. You need to add a simple scale, as shown in Figure 25, so that you can measure the angles. A strip of white paper, with dark lines drawn at intervals of 0.5 cm, should do the trick, and you can fix this to the card screens with Blu-tack. By positioning a double slit correctly in front of your eye you will be able to see the various bright diffracted beams that it produces, and at the same time you will also be able to see the scale through

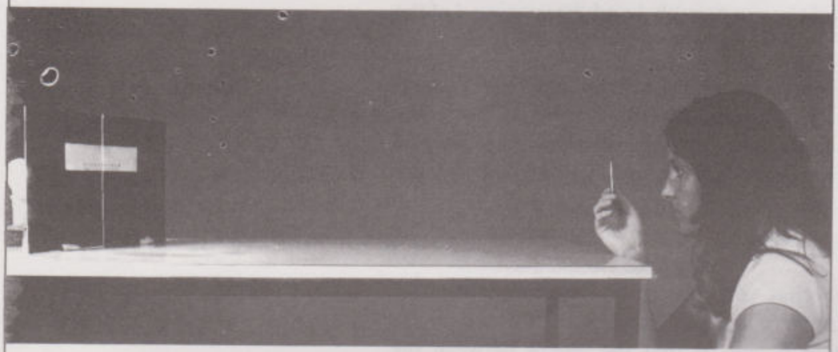


FIGURE 25 The arrangement suggested for measuring the wavelength of light. Remember that the double slit must be parallel to the slit that is in front of the light bulb.

the transparent area above the slits. Since the angles are all very small, the best procedure to adopt is probably to record the apparent scale positions from which the second or third beams on either side of the source appear to come. To work out the angles you will also need to measure the distance between the scale and the double slit; the way that the angles and distances are related is shown in Figure 26. From your measurements you should be able to work out the value of $\sin\theta$ that corresponds to a certain integer n, and then, using the value of d marked underneath the double slit, you can calculate the wavelength of light from equation 6. You should estimate the errors in the quantities that you measure, and determine the possible error in the wavelength that you calculate. The slit spacings marked on the transparency are accurate to 3 per cent.

Try this experiment now. If you have trouble working out the value of $\sin\theta$ from your measurements, then refer to the comments in Appendix 1, p. 46. When you have calculated a value for the wavelength of light, you should compare your result with the result on p. 47, which was obtained by one of the Course Team members.

So the wavelength of light is really very small—about two million of them will fit into a metre! And this is the reason that we are not normally aware of diffraction of light around corners. As an example, compare the diffraction of water waves by the double slit shown in Figure 4a with the diffraction of light by the same slits. The diffraction angle θ is proportional to the wavelength λ; remember that $\sin\theta = n\lambda/d$ for two slits with separation d. Therefore the angles at which light would be diffracted are 3×10^4 times smaller than the angles at which the water waves are diffracted, since the wavelength of light (5×10^{-7} m) is about 3×10^4

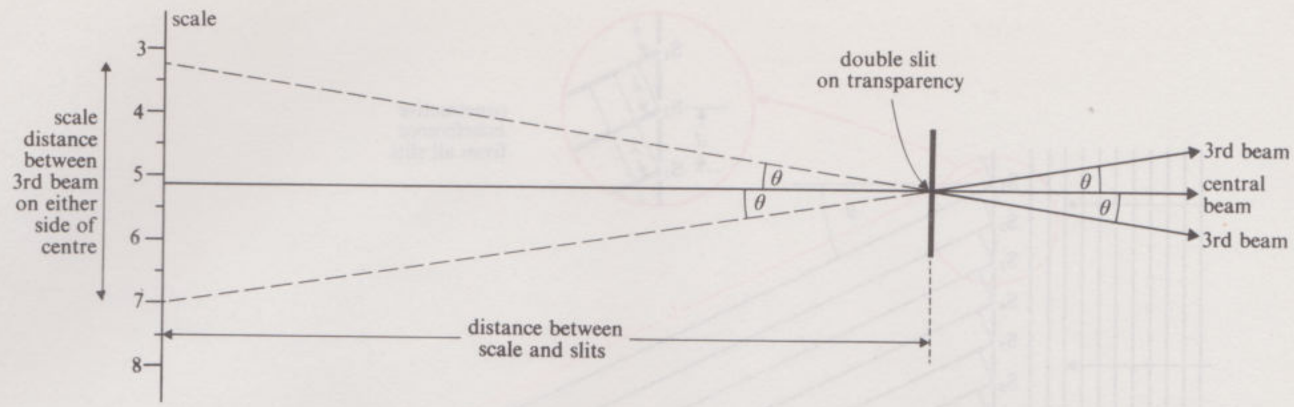


FIGURE 26 The relationship between the diffraction angle θ and the distances measured in Home Experiment 2.

times smaller than the wavelength of the ripples shown in Figure 4a. This means that the angles between the diffracted light beams would only be about 8×10^{-4} degrees, which is much too small to observe.

> What are the period and frequency of light? (Use your experimentally determined wavelength in your calculation.)
>
> $c = \lambda/T$, and so $T = \lambda/c$. Using $\lambda = 5 \times 10^{-7}\,\text{m}$ and $c = 3 \times 10^{8}\,\text{m}\,\text{s}^{-1}$ we get $T = 5 \times 10^{-7}\,\text{m}/3 \times 10^{8}\,\text{m}\,\text{s}^{-1} = 1.7 \times 10^{-15}\,\text{s}$. Rather short! Since $f = 1/T$, the corresponding frequency is $6 \times 10^{14}\,\text{Hz}$.

This estimate of the period should make it clear why we do not see the intensity of a light wave oscillate in a way analogous to the bobbing up and down of a buoy on the surface of water. The period is far, far shorter than our eyes (or any other light detector for that matter) can respond to. In fact our eyes add up the energy incident on each point of the retina over an interval of about 0.05 s, and any variations of energy occurring over shorter intervals will be averaged out. It is this averaging behaviour of the eye that explains why the series of separate pictures we see flashed on a cinema screen at a frequency of 24 Hz gives the impression of continuous motion.

4.7 Diffraction gratings and colour; Home Experiment 3

By investigating the diffraction produced by a double slit, you were able to measure the wavelength of light. We shall now discuss the diffraction produced by an object known as a diffraction grating, and then an experimental observation will allow you to reach some conclusions about the differences between light of various colours.

diffraction grating

A *diffraction grating* is simply a large number of identical parallel slits that are equally spaced, with a separation d. If we illuminate such a grating with parallel waves (as shown in Figure 27) then each slit will radiate light in the same way as the single slit that was shown in Figure 22. If we had drawn in the circular wave crests centred on each slit, Figure 27 would be an undecipherable mess, so instead we have used the approach that was used in the inset of Figure 24. Lines are drawn from each slit to a distant point, and, because the point is far away, the lines are all approximately parallel.

> What is the condition that the angle θ to the distant point must obey if the waves from two *adjacent* slits are to interfere constructively?
>
> $\sin\theta = n\lambda/d$. Naturally it is exactly the same condition as for a double slit, since two adjacent slits are equivalent to a double slit with spacing d.
>
> What is the condition for waves from *all* slits to interfere constructively?
>
> $\sin\theta = n\lambda/d$ again. If the wave from each slit interferes constructively at a certain angle with the wave from an adjacent slit, then *all* waves must interfere constructively at the same angle.

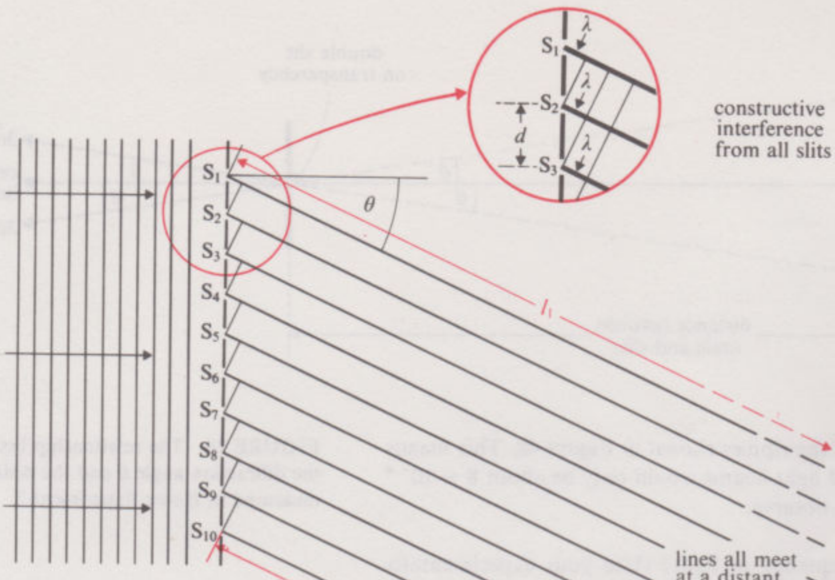


FIGURE 27 Diffraction by a grating is similar in principle to diffraction by two slits. For simplicity only ten slits have been shown, and each is oriented perpendicular to the plane of the diagram. Constructive interference occurs at the angle θ shown in this diagram, as you can see from the magnified inset.

This may be clearer if you consider the differences in the distances l from the various slits to the distant point, as shown in Figure 27. Take the case of $n = 1$ as an example. You should remember from the discussion of the double slit that this means that $l_1 - l_2 = \lambda$, $l_2 - l_3 = \lambda$, $l_3 - l_4 = \lambda$, and so on, where $l_1, l_2, l_3 \cdots$ are the distances from slits 1, 2, 3 $\cdots$ to the distant point. Thus $l_1 - l_3 = 2\lambda$, $l_1 - l_4 = 3\lambda$, and $l_1 - l_{10} = 9\lambda$. Note that each of these differences is an integral number of wavelengths. Therefore waves from every slit must arrive at the distant point in step and so all of these waves will interfere constructively.

The waves from adjacent slits interfere *destructively* at a distant point if

$$\sin\theta = (n + \tfrac{1}{2})\lambda/d$$

just as in the case of a double slit. Thus when $\sin\theta = \frac{1}{2}\lambda/d$, the wave from slit 1 cancels that from slit 2, since $l_1 - l_2 = \lambda/2$ as we have shown in Figure 28a. The wave from slit 3 cancels that from slit 4 (since $l_3 - l_4 = \lambda/2$), and so on, so complete destructive interference takes place at the distant point. However, this does not mean that you would see exactly the same thing if you looked through a double slit and through a diffraction grating with the same slit spacing d. As well as getting complete destructive interference at angles at which waves from adjacent slits cancel, as shown in Figure 28a, we also get complete destructive interfer-

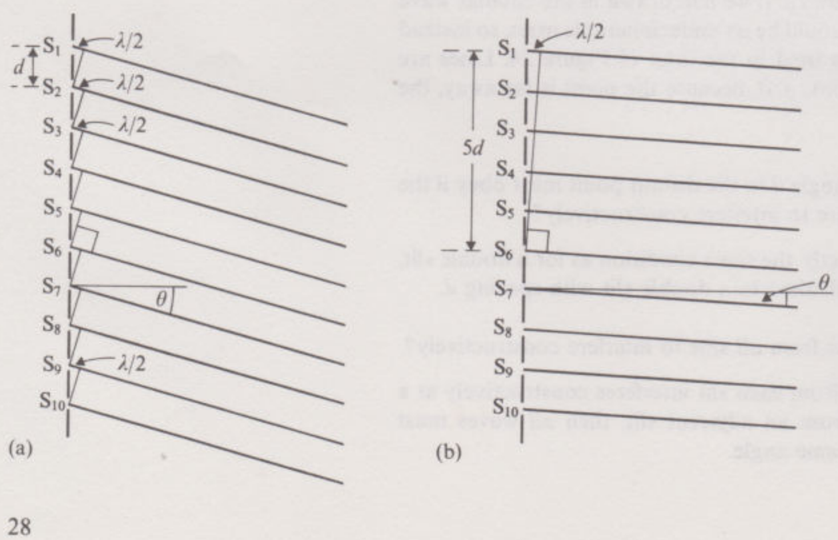


FIGURE 28 Destructive interference occurs at a distant point when the waves arriving from all slits cancel. Two possible ways in which this can happen are shown. In (a) the waves from adjacent slits cancel, and in (b) the waves from slits that are $5d$ apart cancel.

ence at angles at which, for example, the waves from slits 1 and 6 cancel, as shown in Figure 28b. For if waves from slits 1 and 6 cancel, then the waves from slits 2 and 7 must also cancel, those from 3 and 8 cancel, and so on, so that there is total cancellation of the waves from all slits at this angle. The same argument that was used for slits 1 and 6 can also be applied to other combinations, such as 1 and 3, 1 and 4, 1 and 5. For each of these pairs, the angle at which cancellation occurs is very slightly different. Though we will not prove this result, it can be shown that destructive interference occurs over a wider range of angles between the angles of constructive interference than is the case for the double slit.

The effect that this has on the diffraction pattern is shown in Figure 29: compare the patterns shown there for a double slit and for gratings with 10 lines and 100 lines.

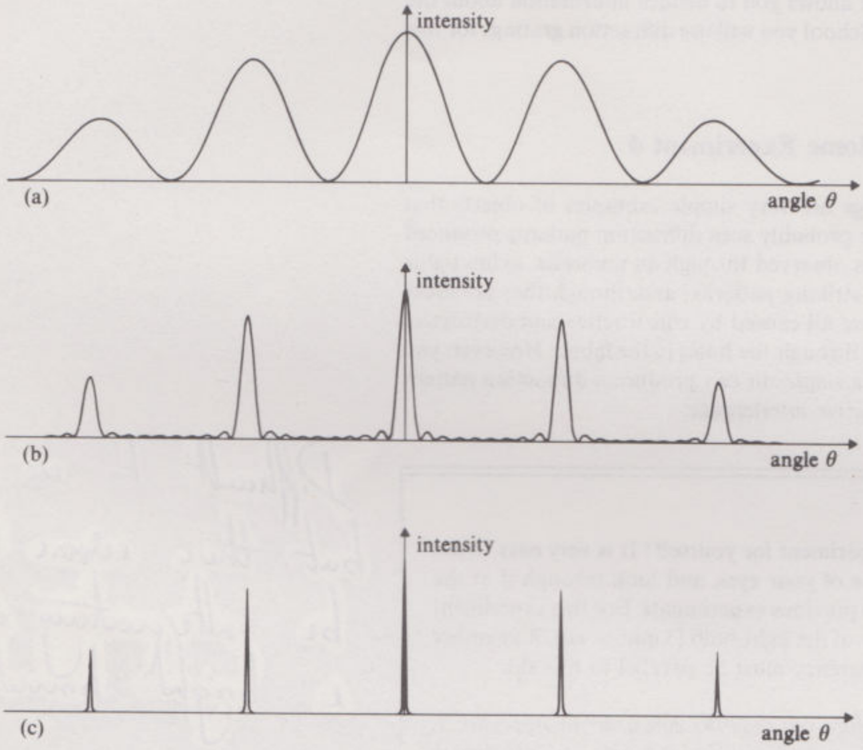


FIGURE 29 A comparison between the diffraction patterns produced by (a) a double slit, (b) a grating with 10 slits, and (c) a grating with 100 slits. Note that the angles at which constructive interference (that is, high intensity) occurs become much more sharply defined as the number of slits increases.

Home Experiment 3

At this stage you should see for yourself the pattern produced by a diffraction grating. There is a grating on the same 35 mm transparency as the double slits, and its slit spacing is 0.02 mm. This is a factor of four smaller than the spacing of the closest double slits provided in your kit, and you should therefore expect the angles at which constructive interference occurs to be about four times larger (remember $\sin\theta \propto 1/d$). The slits in the diffraction grating are too small to be seen with the naked eye. They are parallel to the long dimension of the transparency, and they should be held so that they are parallel to the slit which is in front of the light bulb. So have a look through the diffraction grating at the light source that you used for the earlier experiments. *Try to explain the colour effects that you observe before reading the comments on this experiment which you will find in* *Appendix 1, p. 47.*

What are the approximate frequencies corresponding to blue light and red light? (Use the values for wavelengths given in the comments on the Home Experiment on p. 47.)

For the blue light, $f \approx 7.5 \times 10^{14}$ Hz; for red light $f \approx 4.3 \times 10^{14}$ Hz. These values are obtained by substituting the values for the wavelength given on page 47 into the equation $f = c/\lambda$ (which is a rearranged form of the equation $c = f\lambda$). Thus for blue light, $\lambda \approx 4 \times 10^{-7}$ m and $c = 3 \times 10^{8}\ \mathrm{m\,s^{-1}}$, and so

$$f = \frac{c}{\lambda} = \frac{3 \times 10^{8}\ \mathrm{m\,s^{-1}}}{4 \times 10^{-7}\ \mathrm{m}} = 7.5 \times 10^{14}\ \mathrm{Hz}.$$

Note that blue waves have a shorter wavelength but a higher frequency than red waves.

The ability of a diffraction grating to separate the different wavelength components of light is widely used. In Units 10 and 11 you will see that a knowledge of the wavelengths that are present in light allows you to deduce information about the source of the light, and at Summer School you will use diffraction gratings for this purpose.

4.8 Diffraction by a single slit; Home Experiment 4

Double slits and diffraction gratings are very simple examples of objects that diffract light strongly, but you have probably seen diffraction patterns produced by many other objects. Street lamps observed through an umbrella, nylon tights or a fine net curtain produce quite striking patterns; and, though they are more difficult to analyse, these patterns are all caused by constructive and destructive interference of the light transmitted through the holes in the fabric. However, you may find it difficult to believe that a *single* slit can produce a diffraction pattern that shows clear evidence of destructive interference.

Home Experiment 4

Seeing is believing, so try the experiment for yourself! It is very easy to do. Hold the single slit* close to one of your eyes, and look through it at the light source that you used for the previous experiments. For this experiment you can use a wider slit in front of the light bulb (3 mm or so). Remember that the single slit on the transparency must be parallel to this slit.

Try this experiment now, and before you read the comments in Appendix 1, p. 47, spend a few minutes thinking about how a single slit can give rise to destructive interference.

4.9 The size of the source

We will conclude this discussion of diffraction, by explaining very briefly why you had to restrict the size of the light source used in your diffraction experiments. The reason should be clear if you consider what would happen if the cardboard screens were removed (try it for yourself experimentally, if you like). Each point

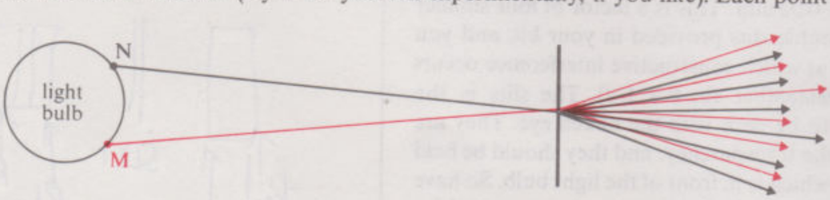


FIGURE 30 The points labelled M and N on the large light bulb give rise to constructive interference in different directions, as indicated by the red and black arrows.

on the light bulb will produce a diffraction pattern that appears to be centred on that point. We have shown this for just two points in Figure 30, and the red and black arrows in that diagram indicate the directions of constructive interference due to these two points. These directions do not coincide, and the

* The single slit is on the left-hand side of the transparency, below the 0.080 mm double slit.

situation will be much worse when all possible source points are considered. The result will be that the pattern blurs out altogether, and directions of constructive and destructive interference cannot be seen.

4.10 Objectives of Section 4

Having studied Section 4, you should be able to do the following:

(a) Apply the principle of superposition to simple diffraction problems.

(b) Describe the appearance of the diffraction patterns produced by a double slit, a diffraction grating and a single slit (Home Experiments 1–4).

(c) Describe a way of measuring the wavelength of light (Home Experiment 2).

(d) Recall that the condition for constructive interference with a double slit or diffraction grating is:

$$\sin\theta = n\lambda/d$$

and use this condition to solve various problems.

(e) Explain why diffraction patterns are not observed with all light sources.

To test your achievement of these Objectives, try the following SAQs.

SAQ 9 Figure 31 shows parallel waves striking a double slit. Using the information in the diagram, determine whether constructive interference, destructive interference or an intermediate type of interference occurs at points A–D.

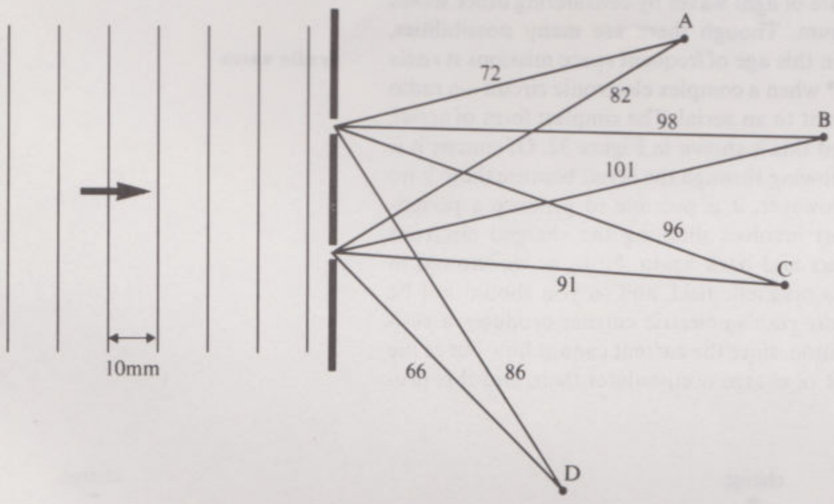


FIGURE 31 For use with SAQ 9. The distances in millimetres between the points and the slits are indicated by the numbers beside the lines.

SAQ 10 Parallel water waves with a wavelength of 20 mm strike a screen perpendicularly. The screen contains two narrow slits that are separated by 70 mm. At what angles do strong beams travel away from the slits?

5 What kind of wave?

You may have been surprised that we could advance so far in understanding interference and diffraction of light without recourse to any more basic theory than that 'light behaves like a wave'. Our detailed and quantitative explanations of diffraction patterns were reached essentially by analogies with water waves, and diffraction by objects that are much more complex than slits can be explained in exactly the same way. To explain or predict diffraction effects, it is not necessary to know the *kind* of wave that light is—only the fact that its behaviour is *wave-like*.

However, we shall now tackle the question 'What kind of wave is light'? We have discussed or mentioned a variety of types of wave in this Course so far. One thing

that all of them have in common, apart from light waves, is that they require a material of some kind to travel through. Seismic waves travel through the solid and liquid regions of the earth, sound waves travel through gases, liquids or solids, and water waves travel on the surface of water. Of course, light can also travel through certain solids, liquids and gases, but, unlike the other waves just mentioned, *it can also travel through empty space*, that is, through a vacuum.

What everyday evidence do you have that light travels through a vacuum?

Sunlight; the light from the Sun reaches Earth after travelling about 150 million kilometres through space, which is a very good vacuum.

This ability to travel through a vacuum provides a first clue to the kind of wave that light is. In all waves there is a parameter that changes with time and position as the wave travels along. In water waves it is the height of the surface that varies periodically; in sound waves it is the compression in the material. It is clear that whatever the parameter that is varying in a light wave with a wavelength of about 5×10^{-7} m and frequency of 6×10^{14} Hz, it must be a parameter that has some meaning in a vacuum. Obviously height of a surface or compression of a material are not suitable parameters.

What effects, or forces, have you come across so far in this course which can act in a vacuum?

Gravity is the most obvious example: the planets orbit around the sun in the vacuum of interplanetary space, and it is gravitational forces that are responsible for keeping them in these stable orbits. However, both magnetic and electric forces can also act in a vacuum.

radio waves

You can get a second clue to the nature of light waves by considering other waves that can propagate through a vacuum. Though there are many possibilities, probably the most obvious example in this age of frequent space missions is *radio waves*. This type of wave is produced* when a complex electronic circuit—a radio transmitter—supplies an electric current to an aerial. The simplest form of aerial, a dipole, is just two straight wires and this is shown in Figure 32. Of course, it is impossible to have a steady current flowing through the aerial because there is no connection between the two ends. However, it is possible to produce a periodically varying current; such a current involves shunting the charged electrons from one end of the aerial to the other and back again. Now, as we showed in Unit 5, an electric current produces a magnetic field, and so you should not be surprised by the fact that a *periodically varying* electric current produces a *periodically varying* magnetic field. In addition, since the current cannot flow out of the ends of the dipole, a varying amount of charge accumulates there and this produces a *varying electric field***.

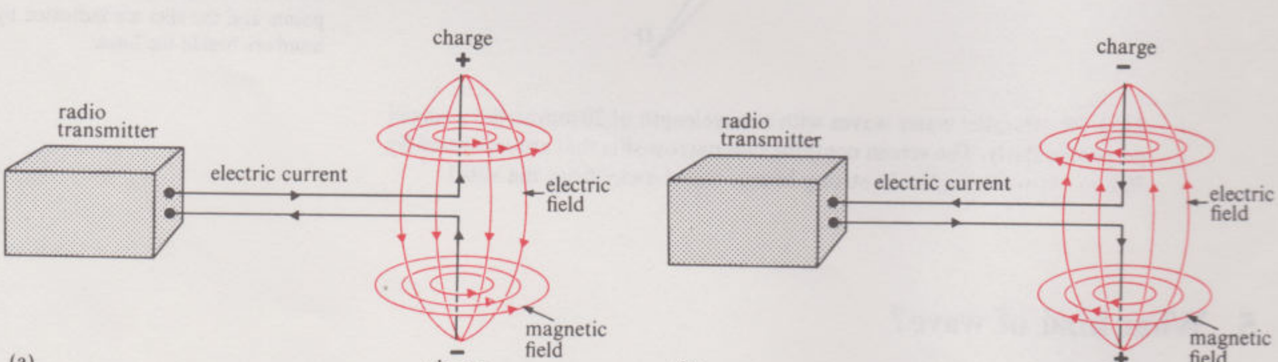


FIGURE 32 A varying electric current in the dipole aerial produces varying magnetic and electric fields. The currents and fields are shown at two instants that are separated by half of the period.

* You are not expected to remember the description of radio-wave generation that follows.

** You have not yet been introduced to electric fields in this Course. For present purposes we can explain them by analogy with magnetic fields. Just as the magnetic field at a point tells you the magnitude and direction of the force that would be exerted on a magnet if it was placed at that point, so the *electric* field tells you the magnitude and direction of the force that would be exerted on a *charged* body.

In Figure 32 you can see a schematic representation of the directions of the magnetic and electric fields at two instants of time that are separated by half of a period.

Now if you periodically waggle the end of a stretched string, waves travel out from your hand along the string, and if you periodically dip your finger into water, surface waves radiate out from the position of your finger. In an exactly analogous way, if a periodically varying current is supplied to an aerial, it produces periodically varying magnetic and electric fields, and both magnetic and electric field waves travel out from the aerial. These waves, radio waves, are therefore called *electromagnetic waves*. In such waves it is the magnetic and electric fields that vary periodically with time and position as the wave travels along.

electromagnetic waves

The way that the electric and magnetic fields in a typical VHF (*V*ery *H*igh *F*requency) radio wave vary with time at a certain position (for example at your radio receiver) are shown in Figures 33a and 33b. The frequency of these waves is identical to the frequency at which the current supplied by the transmitter varies. Also shown, in Figures 33c and 33d, are the ways that the fields at a certain instant vary with distance from the source of the waves. For most efficient generation of radio waves, the length of the dipole is equal to one-half of the wavelength of the radiation produced. The similarity between these graphs and those shown earlier for water waves is readily apparent. We need only substitute 'height of water surface' for 'electric field' or 'magnetic field', and change the magnitudes of the wavelength and period, and the graphs shown in Figure 33 would describe the propagation of a water wave.

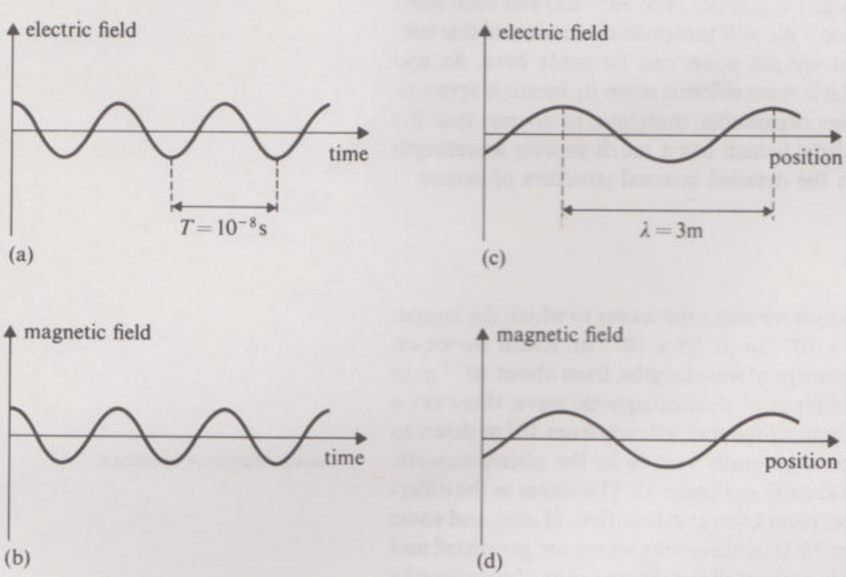


FIGURE 33 Graphs representing 3 m wavelength radio waves in air. (a) and (b) show the variation with time of electric and magnetic fields at a certain position; (c) and (d) show the variation with position of electric and magnetic fields at a certain instant of time.

Now you may be thinking that light and radio waves appear to have nothing in common apart from the ability to travel through a vacuum. Their wavelengths certainly are very different—by a factor of about 10^7. But how do their velocities compare?

Using the data in Figure 33, can you deduce the value of the velocity of radio waves in air?

The velocity is $3 \times 10^8\,\mathrm{m\,s^{-1}}$. The period and wavelength of a particular radio wave are shown in Figure 33, and the velocity is found by substituting these values into equation 2: $v = \lambda/T$.

How does this compare with the velocity of light?

It is the same to within the accuracy with which we have quoted the values here and on p. 8.

In fact the best measurements of these velocities, which are accurate to about one part in a million, do not show any difference between the velocity of light and the velocity of radio waves. This cannot be mere coincidence. It is very strong evidence that *light is also an electromagnetic wave*, and that its wave-like nature

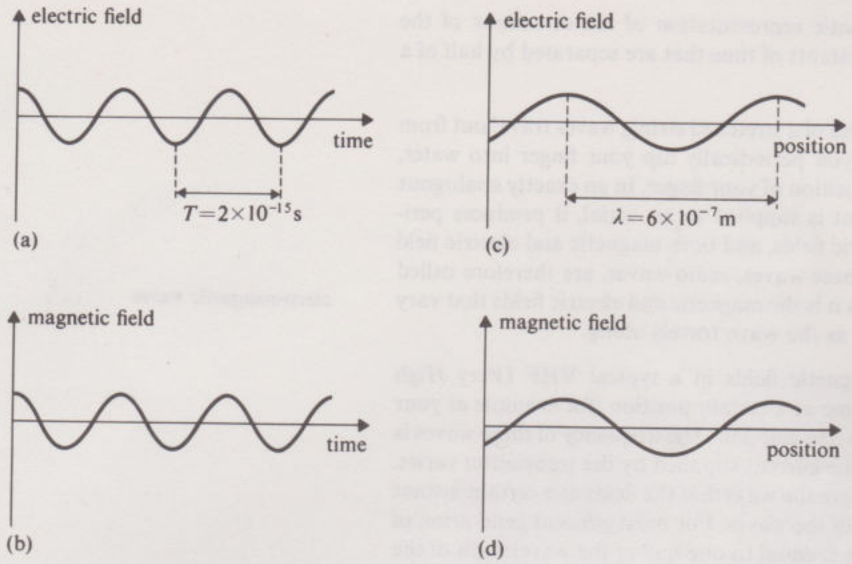


FIGURE 34 Graphs representing the electric and magnetic field variations in a light wave travelling through air. (a) and (b) show the variation with time of the fields at a certain position; (c) and (d) show the variation with position of the fields at a certain instant of time.

can be represented graphically in the way shown in Figure 34. These graphs are the same as those shown in Figure 33 except that the magnitudes of the wavelength and the period have been changed.

The identification of light as a form of electromagnetic wave raises the following question. How can waves with such high frequencies (6×10^{14} Hz) and such short wavelengths (5×10^{-7} m) be produced? We will postpone discussion of this subject until Units 10 and 11, but one simple point can be made here. As was mentioned earlier, a dipole radio aerial is most efficient when its length is approximately half of the wavelength. It seems reasonable, therefore, to assume that the processes responsible for generating light (which has a much shorter wavelength than radio waves) must be sought in the detailed internal structure of matter.

5.1 The electromagnetic spectrum

electromagnetic spectrum

The wavelengths of visible light, by which we mean the waves to which the human eye is sensitive, range from about 4×10^{-7} m to 7.5×10^{-7} m. Radio waves, on the other hand, cover a much greater range of wavelengths, from about 10^{-1} m to 1 000 m. But in addition to these two types of electromagnetic wave, there are a variety of others that span a wavelength range that extends from 10^3 m down to 10^{-14} m. This whole range of waves is generally known as the *electromagnetic spectrum*, and is displayed diagrammatically in Figure 35. The waves in the different regions of the electromagnetic spectrum have a wide variety of uses, and some of these have been indicated in Figure 35. In addition the waves are generated and detected in different ways. However, in spite of the immense range of wavelengths and frequencies involved, all of the waves are essentially similar. All are electromagnetic waves, and all travel through air with the same velocity as light—$3 \times 10^8\,\mathrm{m\,s^{-1}}$.

5.2 Objectives of Section 5

Having studied Section 5, you should now be able to do the following:

(a) Explain what is meant by an electromagnetic wave.

(b) Give reasons that support the conclusion that light is an electromagnetic wave.

(c) Place in order the various regions of the electromagnetic spectrum according to their frequency or wavelength.

(d) Recall the orders of magnitude of the frequencies or wavelengths of electromagnetic waves from the various regions of the spectrum.

(e) Recall that all electromagnetic waves travel at a velocity of $3 \times 10^8\,\mathrm{m\,s^{-1}}$ in air.

To test your achievement of these Objectives try the following SAQs.

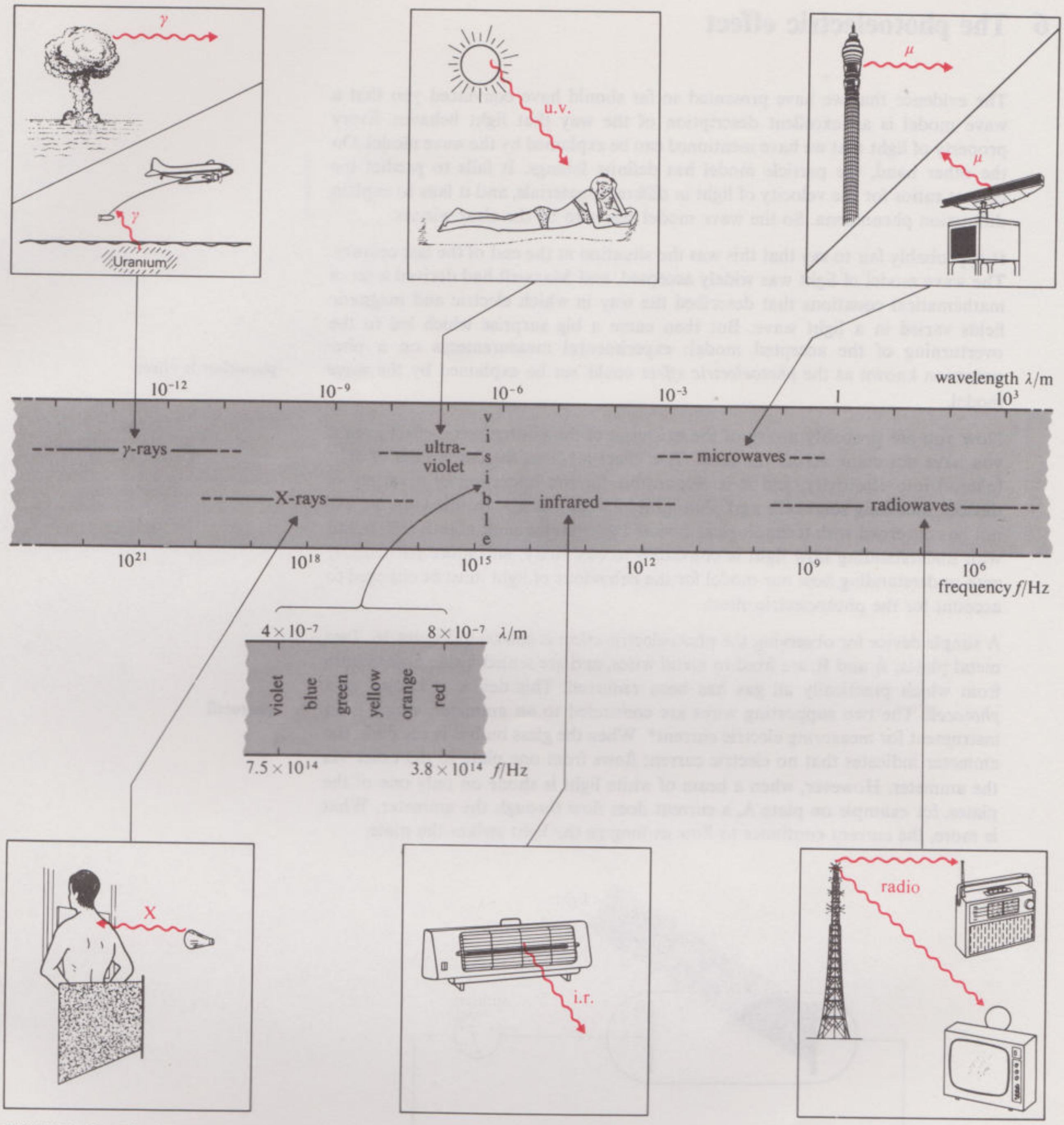


FIGURE 35 The electromagnetic spectrum and some of its uses. Note that the boundaries between the different regions are not well defined.

SAQ 11 In a science-fiction novel a ray gun is described that produces rays with a wavelength of 10^{-11} m and frequency of 3×10^{22} Hz. Could these rays be part of the electromagnetic spectrum?

SAQ 12 Sketch graphs that illustrate the variation of electric and magnetic fields produced by ultraviolet waves (a) as a function of position at a certain instant, (b) as a function of time at a certain position. Make use of data from Figure 35 to indicate the magnitudes of wavelength and frequency on your sketches.

SAQ 13 What are the names usually given to electromagnetic waves with the following values of wavelength or frequency?

(i) $\lambda = 5 \times 10^{-2}$ m
(ii) $f = 2 \times 10^{21}$ Hz
(iii) $\lambda = 4 \times 10^{-7}$ m
(iv) $f = 4 \times 10^{13}$ Hz
(v) $\lambda = 10^{-8}$ m

6 The photoelectric effect

The evidence that we have presented so far should have convinced you that a wave model is an excellent description of the way that light behaves. Every property of light that we have mentioned can be explained by the wave model. On the other hand, the particle model has definite failings. It fails to predict the correct ratios for the velocity of light in different materials, and it fails to explain diffraction phenomena. So the wave model seems to be the clear winner.

It is probably fair to say that this was the situation at the end of the last century. The wave model of light was widely accepted, and Maxwell had derived a set of mathematical equations that described the way in which electric and magnetic fields varied in a light wave. But then came a big surprise which led to the overturning of the accepted model: experimental measurements on a phenomenon known as the *photoelectric effect* could *not* be explained by the wave model.

photoelectric effect

Now you are probably aware of the existence of the photoelectric effect even if you have not come across the term. This effect involves the conversion of light (*photo-*) into electricity, and it is responsible for the operation of a variety of devices, including solar cells and photographic light meters. In this Unit we will not be concerned with technological devices based on the photoelectric effect, but with understanding how light is converted to electricity, and more particularly with understanding how our model for the behaviour of light must be changed to account for the photoelectric effect.

A simple device for observing the photoelectric effect is shown in Figure 36. Two metal plates, A and B, are fixed to metal wires, and are sealed inside a glass bulb from which practically all gas has been removed. This device is known as a *photocell.* The two supporting wires are connected to an ammeter, which is an instrument for measuring electric current*. When the glass bulb is in the dark, the ammeter indicates that no electric current flows from one plate to the other via the ammeter. However, when a beam of white light is shone on only one of the plates, for example on plate A, a current does flow through the ammeter. What is more, the current continues to flow as long as the light strikes the plate.

photocell

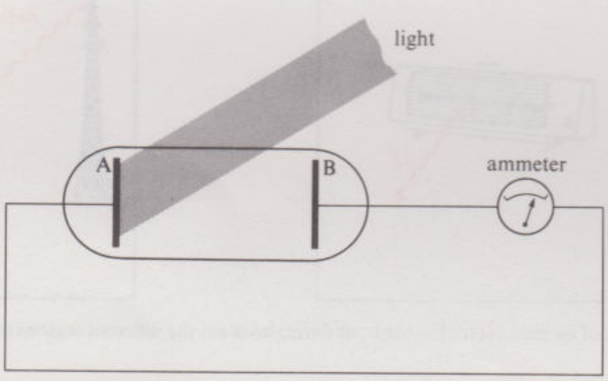


FIGURE 36 A simple photocell connected to an ammeter.

The electric current that passes through a wire is a flow of negatively charged particles called electrons, as we mentioned in Units 5 and 8, and the ammeter indicates that the electrons are travelling from B to A through the connecting wire. The electrons cannot be accumulating at plate A, for if they did they would repel other electrons and stop the current, and so they must be crossing back from plate A to plate B *through the vacuum.* This is unusual behaviour for electrons! The electrons in a metal do not normally 'leak' out of it into the surrounding air (or a vacuum), since there are attractive forces holding them inside. What is happening—and this is the essence of the photoelectric effect—is that energy from the light beam is being transferred to electrons, and is giving these electrons enough energy to escape from the attractive forces holding them inside the metal.

* It is not important at this stage to know the details of how an ammeter works. It is sufficient to say that its operation generally depends on the forces between currents and magnets.

The escaping electrons can travel from A to B, and then through the connecting wire and ammeter, and back to plate A. Obviously the current will continue to flow as long as the light is providing the energy required for electrons to escape from the illuminated plate.

As you might expect, a more intense beam of white light produces a larger current because the energy carried by a light beam increases as the beam intensity is increased. However, much less predictable behaviour is observed when *monochromatic* light is used rather than white light. Monochromatic (literally one-colour) light is composed of a very narrow range of frequencies, whereas white light is generally composed of frequencies spanning the whole range of the visible spectrum. No current at all is observed if the frequency of the monochromatic light is below a certain *threshold value* f_t; a photoelectric current is only produced if the frequency is greater than f_t. What is more, although very high intensity light beams do not produce any photoelectric current if their frequency is below the threshold frequency f_t, even the lowest intensity beam will produce some current if the light has a frequency greater than f_t.

monochromatic light

threshold frequency, f_t

These observations seem to be incompatible with the simple wave model that we have assumed so far in this Unit. As you saw in Section 4.3, the energy of a wave does not depend on its frequency, but rather on the square of its amplitude. A simple wave model that is based on our experience of how water waves behave would therefore predict the same photoelectric current at all frequencies.

Another fact that our wave model cannot readily explain is that the photoelectric current starts to flow as soon as the light is switched on, irrespective of the intensity of the light. This obviously means that electrons must be ejected from the metal as soon as the light strikes it. Now, an electron must absorb a certain amount of energy if it is to overcome the attractive force that holds it inside the metal. The lower the intensity of the light waves, the longer will be the time required by an electron to pick up enough energy from the waves so that it can escape. For example, a rough calculation indicates that if a photocell is placed one metre away from a light bulb that emits one watt of light at frequencies greater than the threshold frequency, then it will take about *a minute* for an electron to absorb enough energy to escape. This delay will be even longer if the light intensity reaching the photocell is lower. This predicted delay is clearly at variance with the experimental observations of instantaneous current flow at all light intensities.

Since it is difficult to visualize what is happening in the photoelectric effect using a wave model, we shall re-open the question of what is the best description of light. It is true that the wave model has explained all the other effects that we have discussed, but the photoelectric effect has exposed the limitations of this model. We will therefore look at other aspects of the photoelectric effect in order to devise a more acceptable model.

6.1 The energy of the photoelectrons

The combination of photocell and ammeter shown in Figure 36 is not suitable for probing more deeply the mysteries of the phótoelectric effect. The limitation of this arrangement is that it is only possible to measure the total current and this only tells us the number of photoelectrons produced per second. However, if a battery is connected in the circuit, then it is possible to measure the energies of the photoelectrons and this additional information points to the way in which energy is transferred from the light beam to the electrons.

When the battery is connected to the photocell in the way shown in Figure 37, a voltage difference V is created between plates A and B. Plate B is at a higher voltage than plate A, and the difference in voltage is equal to the voltage of the battery. When a beam of monochromatic light strikes plate A, electrons will be ejected in exactly the same way as when the battery is not used. However, as these electrons travel towards plate B, which is at a higher voltage than plate A, their electrical energy must change.

> Bearing in mind that electrons are negatively charged, do you think the electrical energy of an electron will increase or decrease as it moves towards plate B?

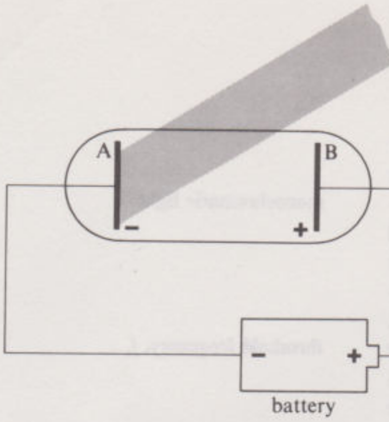


FIGURE 37 A battery included in the photocell circuit, with its negative terminal connected to the illuminated plate of the photocell.

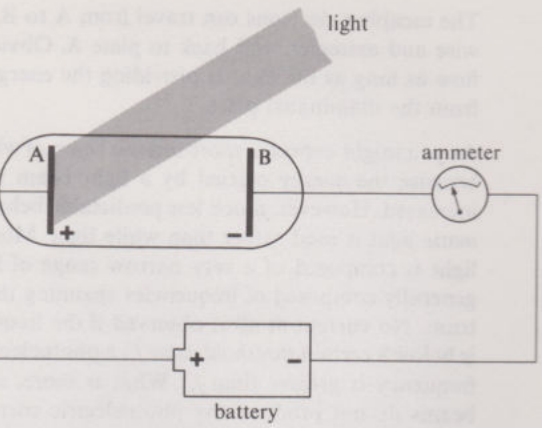


FIGURE 38 The same situation as shown in Figure 37, except that the positive terminal of the battery is connected to the illuminated plate.

It decreases. In Unit 8 we pointed out that the electrical energy of a charge Q at a point with voltage V is just QV. B is at a higher voltage than A, but the charge on an electron is negative and is usually denoted by $-e$. So the electrical energy of an electron is smaller at B than at A by an amount eV.

Into what form of energy is the lost electrical energy converted?

Kinetic energy. The decrease in electrical energy of each electron is compensated for by an equal increase of kinetic energy. So each electron accelerates as it moves towards plate B, and it reaches this plate with an amount of kinetic energy that is eV greater than it had when ejected from plate A. This is yet another example of the conservation of energy.

A simple gravitational analogy of this would be a footballer on a perfectly smooth hillside, who is kicking balls downhill along the ground to a friend. As the ball rolls downhill it loses gravitational energy and its velocity increases. However hard the footballer kicks the ball, the kinetic energy of the ball will increase by an amount mgh between the time it leaves his foot and the time it reaches his friend, (where m is the mass of the ball, and h is the difference in height between the points on the hill where the two players are standing).

Now consider what happens when the battery is connected the opposite way to the photocell, that is, with the positive terminal to plate A and the negative terminal to plate B, as shown in Figure 38. The electrical potential energy of an electron is now *higher* at plate B than at plate A.

What happens to the photoelectrons in this situation? Do all ejected electrons reach plate B?

Since the electrical potential energy of the electrons increases as they travel from A to B, their kinetic energy must decrease. They slow down. Some will stop before reaching B, and then will be accelerated back towards plate A.

If the voltage difference between plate A and plate B is V, then only electrons ejected from plate A with a kinetic energy that is greater than eV will reach plate B; all electrons with less kinetic energy than this will be turned back to plate A. It is exactly analogous to kicking a ball uphill: the ball only gets to a point that is at a height h further uphill if it starts out from the footballer's boot with an initial kinetic energy that is greater than mgh.

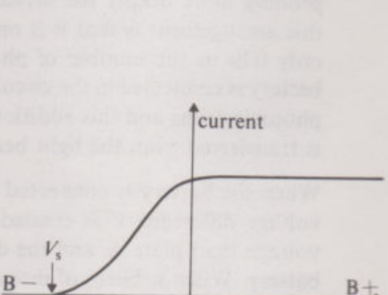


FIGURE 39 The current produced by the photocell depends on the battery voltage. To the right of the vertical axis, plate B is at a positive voltage with respect to A. To the left, plate B is at a negative voltage with respect to A.

You can see in Figure 39 the way that the photoelectric current changes when batteries with different voltages are connected to the photocell (while a monochromatic beam of light of constant intensity falls on plate A). You are probably not surprised to see that the current is always the same as long as plate B is at a positive voltage with respect to plate A. The current is determined by the rate at which electrons arrive at plate B; this is always equal to the rate at which the electrons are ejected from plate A (since all ejected electrons reach plate B when it is at a positive voltage), and this is determined by the frequency and intensity of the light and not by the voltage. On the other hand, when a negative voltage is

applied to plate B the current is smaller, since some electrons are not ejected with enough energy to reach plate B. The more negative the voltage of plate B, the smaller is the proportion of electrons with sufficient energy to reach plate B and the smaller is the current. In fact *all* of the electrons ejected from plate A are stopped before they reach plate B when this negative voltage has a magnitude greater than a certain value V_s known as the *stopping voltage*, and so no current is observed at all.

stopping voltage, V_s

Can you deduce the *maximum* kinetic energy of the ejected electrons from this observation?

The current is 'turned off' when the voltage at plate B is V_s lower than at plate A, so the electrical energy of an electron at plate B is eV_s higher than at plate A. The *maximum* kinetic energy of the ejected electrons must therefore be eV_s, since electrons with this kinetic energy will just be able to climb the electrical energy hill between plates A and B.

The results shown in Figure 39 are obtained with monochromatic light of a particular frequency. When the experiment is repeated using monochromatic beams of other frequencies, it turns out that the magnitude of the stopping voltage V_s becomes larger as the frequency is increased. The type of results that are obtained are shown in Figure 40, where we have plotted the maximum kinetic energy of the photoelectrons eV_s for various frequencies f. Below the threshold frequency f_t no photoelectric effect occurs at all, as we pointed out earlier. Light that has a frequency just above f_t produces photoelectrons with very low kinetic energy, and they can be stopped with very small voltages. Light that has a higher frequency produces electrons with higher kinetic energies, and these require higher voltages to stop them.

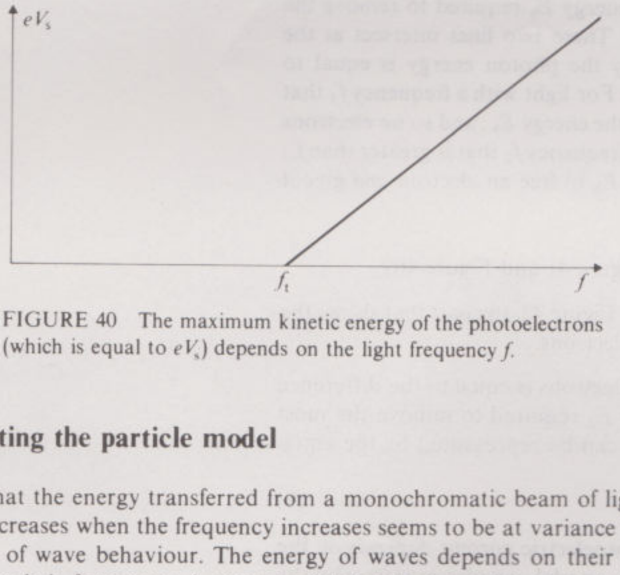


FIGURE 40 The maximum kinetic energy of the photoelectrons (which is equal to eV_s) depends on the light frequency f.

6.2 Resurrecting the particle model

The fact that the energy transferred from a monochromatic beam of light to an electron increases when the frequency increases seems to be at variance with our experience of wave behaviour. The energy of waves depends on their intensity rather than their frequency. A wave model therefore predicts that the number and/or energy of the photoelectrons increases when the *intensity* increases, but that the number and energy of the photoelectrons is independent of the *frequency*. Since this latter prediction is incorrect, we are reopening the question of what is the best model for the behaviour of light.

So let us come back to a particle model. If we assume that monochromatic light of frequency f is made up of identical particles, *each carrying energy hf* (where h is a constant that can be determined experimentally), then we can account for the photoelectric experiments very simply. The particles of light, generally known as *photons*, collide with electrons in the metal. The energy hf carried by a photon is transferred to an electron, and the photon disappears—it is annihilated, if you like. Now when the frequency of the light is less than the threshold frequency f_t, the energy carried by the photon is insufficient to free even the most weakly bound electrons from the metal, and all of the photon energy is converted to heat. However, when the frequency of the light is greater than f_t, the energy of each photon is sufficient not only to eject one of these electrons, but also to give it some kinetic energy as well. And the higher the photon energy (that is, the higher the frequency of the light), the higher will be the kinetic energy of the photoelectron.

photon, energy = hf

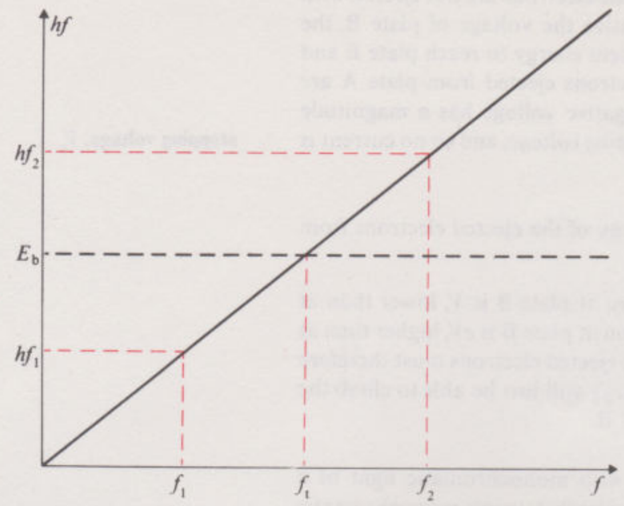


FIGURE 41 The relationship between photon energy hf and light frequency f (continuous line). The broken black horizontal line represents the smallest amount of energy needed to remove an electron from the metal plate.

All of this can be simply related to Figure 41. The continuous line on this graph represents the relationship between the photon's energy and its frequency:

$$E = hf \tag{8}$$

and the broken horizontal line represents the energy E_b required to remove the most weakly bound electrons from the metal. These two lines intersect at the threshold frequency f_t, since at that frequency the photon energy is equal to the energy that binds the electrons in the metal. For light with a frequency f_1 that is less than f_t, the photon energy hf_1 is less than the energy E_b, and so no electrons are ejected. On the other hand, for light with a frequency f_2 that is greater than f_t, the photon energy hf_2 can provide the energy E_b to free an electron *and* give it some kinetic energy as well.

Can you see the relationship between Figure 41 and Figure 40?

Figure 40 is essentially the top half of the Figure 41, the part that shows the maximum kinetic energy of the ejected electrons.

The maximum kinetic energy E_k of the photoelectrons is equal to the difference between the photon energy hf and the energy E_b required to remove the most weakly bound electrons from the metal. This can be represented by the equation:

$$E_k = hf - E_b \tag{9}$$

As well as accounting for the way that the photoelectric current depends on the frequency of the light, the particle—or photon—model can also explain why the current appears instantaneously even when the intensity of the light is very low. The intensity is just the energy that strikes an area of one square metre in one second. According to our model, the energy carried by each photon in a monochromatic beam is identical, and is equal to hf. Thus a change in intensity must be due to a change in the *number* of photons striking an area of one square metre in one second. In fact we can write the relationship very simply:

intensity = photon energy × number of photons per square metre per second

or number of photons per square metre per second = I/hf.

Therefore since each photon has exactly the same chance to eject an electron, the small number of photons in a low intensity beam will only eject a small number of electrons, but they will be able to do this as soon as they strike the metal.

The constant h is known as *Planck's constant*, after the famous physicist who was concerned with the interaction of light with materials. Since $h = E/f$, the units of h must be the same as the unit of energy divided by the units of frequency:

Planck's constant, h

$$\text{So units of } h \equiv \frac{\text{joule}}{\text{second}^{-1}} = \text{joule second},$$

and this is abbreviated to J s. In these units the actual value of Planck's constant is $h = 6.63 \times 10^{-34}$ joule second (J s).

It is straightforward to calculate the energy of each photon in a monochromatic beam of light. For example, in red light, which has $f = 4.5 \times 10^{14}$ Hz, each photon has an energy given by:

$$E = (6.63 \times 10^{-34}\,\mathrm{J\,s}) \times (4.5 \times 10^{14}\,\mathrm{Hz})$$
$$= 3.0 \times 10^{-19}\,\mathrm{J}$$

In a similar way you should be able to calculate that the energy of each photon in blue light, $f = 6.8 \times 10^{14}$ Hz, is 4.5×10^{-19} joule.

The photon model is equally applicable to other regions of the electromagnetic spectrum. For example, in an X-ray beam that has a frequency of 3×10^{18} Hz, the photons have energy:

$$E = hf = (6.63 \times 10^{-34}\,\mathrm{J\,s}) \times (3 \times 10^{18}\,\mathrm{Hz})$$
$$= 2.0 \times 10^{-15}\,\mathrm{J}$$

6.3 Objectives of Section 6

Having studied Section 6, you should be able to do the following:

(a) Describe photoelectric experiments and their results, and explain to what extent they can be accounted for by the wave model and the photon model.

(b) Recall that photon energy $E = hf$, and calculate the energy of a photon when given the frequency or vice versa.

(c) Perform simple calculations involving (i) the threshold frequency of photocells, (ii) the conversion of photon energy to the kinetic energy of an electron, (iii) the conversion of kinetic energy to electrical energy.

(d) Recall how the intensity is related to the number of photons, and calculate one of these quantities when the other is specified.

To test your achievement of these Objectives, try the following SAQs.

SAQ 14 Monochromatic light falls on a photocell, as shown in Figure 36 (p. 36). The threshold frequency f_t of this photocell is 5.0×10^{14} Hz. Will a photoelectric current be produced if the light has the following frequencies: (a) 6.8×10^{14} Hz, (b) 5.5×10^{14} Hz, (c) 4.5×10^{14} Hz, (d) 4.0×10^{14} Hz?

SAQ 15 A small ball has a mass of 10^{-5} kg and a charge of $+10^{-9}$ coulomb. It is held midway between two metal plates, one of which is connected to the positive terminal of a 100 volt battery and the other to the negative terminal. Assuming that gravitational forces can be neglected, which plate will the ball strike when it is released, and what will its velocity be just before impact? (You may assume that in its initial position the electrical energy of the ball is midway between the values it would have at the two plates.)

SAQ 16 Rather than releasing the ball described in the previous SAQ from rest, it is given a velocity of $0.2\,\mathrm{m\,s^{-1}}$ towards the positive plate. Will the ball strike the positive plate?

SAQ 17 Monochromatic light with a frequency of 6×10^{14} Hz falls on a photocell that has a threshold frequency of 5×10^{14} Hz. What is the maximum kinetic energy of the ejected electrons?

(*Hint* The electrons with the greatest kinetic energy will be those that were most weakly bound, that is, those that required energy E_b to escape, as shown in Figure 41.)

SAQ 18 The intensity of sunlight reaching the earth's surface on a sunny day is about $10^3\,\mathrm{watt\,m^{-2}}$. Assuming that the average frequency of sunlight is 6×10^{14} Hz, estimate how many photons would strike this opened book (area $\sim 0.1\,\mathrm{m^2}$) each second if you were reading it outside on a sunny day.

SAQ 19 What is the energy of a typical infrared photon?

SAQ 20 List three experimental photoelectric results that can be accounted for more easily by a photon model than by a simple wave model, and explain how the predictions made by a simple wave model differ from the experimental observations.

7 Waves and particles

Our attempts to find a model for the behaviour of light seem to have reached an impasse. The diffraction and interference of light is beautifully explained by a wave model, but cannot be explained by a particle model. Conversely, the photoelectric effect, which is completely accounted for by a particle model, cannot easily be visualized with a simple wave model. It seems that light is not really a wave, nor is it really a collection of particles: neither model is an adequate description since neither model explains *all* of the observed properties of light.

The failure of both wave and particle models, and the fact that no other simple model accounts for the properties of light, will probably worry you at first. However, you should console yourself with the fact that you are not alone; many generations of scientists have puzzled over this enigma, and the members of the S101 Course Team have certainly done so too. The difficulty arises because we are trying to explain light in terms of analogies with phenomena that we observe in the macroscopic, or large-scale, world. We are using mental pictures of tiny billiard balls, or of water waves, to help us visualize why light behaves in the ways observed. These mental pictures are imperfect analogies. In fact there is no macroscopic phenomenon that can be used as a perfect analogy for light. No macroscopic phenomenon exhibits both wave-like and particle-like properties, and so there is no hope of devising a simple model.

Now rather than abandoning both wave and particle models because of their failings, it is much more helpful to combine the strengths of each model. The strength of the wave model is its ability to describe so completely the way that light travels. You have seen how it accounts for the way light travels through a double slit, a diffraction grating and a single slit to produce a variety of diffraction patterns. Indeed, the wave model can account for the diffraction pattern produced by any object, however complex it may be. On the other hand, the strength of the particle, or photon, model is that it can explain the interactions of light with matter. We have only discussed in detail how the photoelectric effect is explained by the absorption of photons, but the detection of light by the retina in the eye, and the interaction of light with photographic film, are also easily accounted for with the photon model. And in addition to explaining the processes in which light is absorbed by matter, the photon model can also explain how light is emitted; this will be one of the important topics in Units 10 and 11. At this point it is sufficient to emphasize that light is always emitted, or absorbed, as a *whole number* of photons. In other words, it is impossible to emit or absorb a fraction of the energy of a photon—it is all or nothing.

How do we combine the best points of the two models? Well, in any situation in which light is *travelling* we use the wave model. In situations in which light is being *absorbed* or *emitted* we use the photon model. This combined approach is often referred to as a *wave–particle duality*.

wave–particle duality

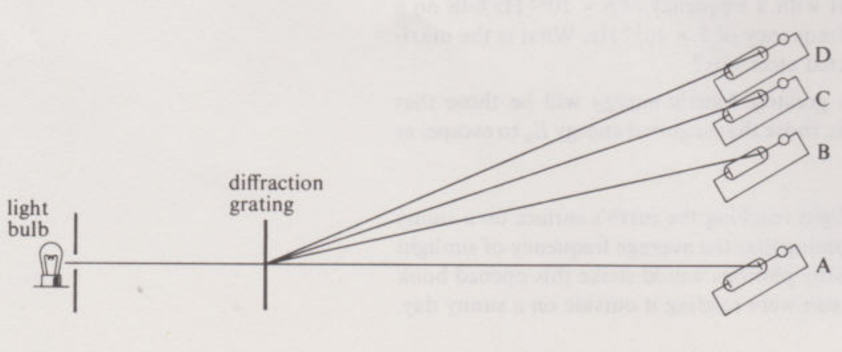


FIGURE 42 Both wave and particle models must be used to predict the outcome of the experiment pictured here, in which light is diffracted by a grating and detected by photocells.

This wave–particle approach can be illustrated by referring to the experiment shown in Figure 42. This is a diffraction grating experiment in which the light travelling in certain directions is detected by a number of photocells. To explain the way that the light travels through the grating, to explain the way that it interferes constructively and destructively as it travels away from the slits, and to explain the appearance of different colours at different angles we must use the wave model. This model will predict the directions in which constructive or destructive interference occurs for the different wavelengths present and, more generally, it will predict the relative intensity of the light at different points. For example, in the experiment in Figure 42, the wave model indicates that all wavelengths interfere constructively at point A, and it might indicate that only the blue wavelengths interfere constructively at B and only the red wavelengths at C, and that all wavelengths interfere destructively at D. Having used the wave model to predict the intensity and wavelengths of light reaching a certain position, the photon model can then explain how the light is absorbed by a photocell at that position. It can explain how an electric current is produced by the photocell, and how the current depends on the wavelength (or frequency) and intensity of the light reaching the photocell. For example, if the photocells in Figure 42 have a threshold frequency in the green region of the spectrum, then the photocell at A will produce a relatively large current (since there is a relatively intense beam containing photons of all colours at that place). A smaller current will be observed at B (since only blue photons reach that position), no current will be observed at C (since the red photons have insufficient energy to eject the electrons), and no current will be observed at D (since no photons reach this point). Thus all experimental observations can be explained if we use waves to model the way that the light travels and photons to model the way that light is absorbed.

The waves and photons are related in two important ways, and these are summarized in Figure 43.

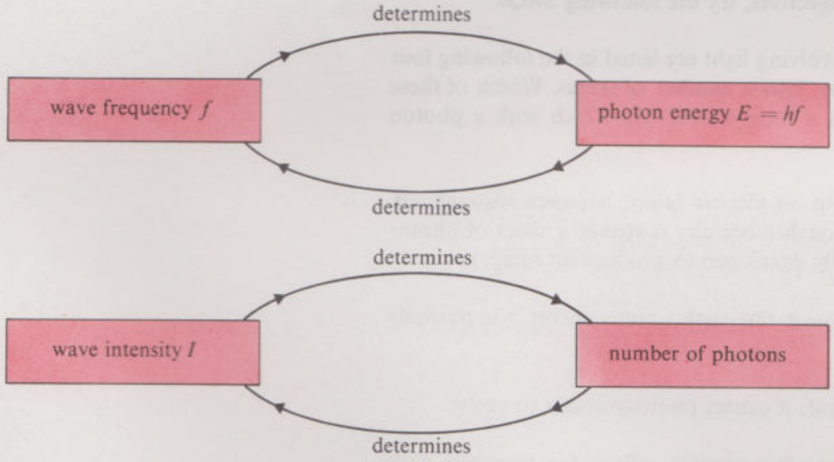


FIGURE 43 Wave–particle duality.

The wave frequency is related to the photon energy, and the wave intensity is related to the number of photons.

7.1 A hybrid model

We have ended up with a model which is neither a wave model nor a particle model, but some combination of the two. By taking some features of the wave model and other features of the particle model, and specifying when each model should be used, we have produced a 'super model' that can account for the properties of light. But this 'super model' does not correspond to any macroscopic phenomenon that is part of our everyday experience, and it is therefore impossible to visualize. All we can do is think in terms of waves in situations where the wave nature of light is important (propagation, diffraction, interference), and think in terms of particles in situations where the particle nature is important (absorption, emission).

We must bear in mind that *light is not a wave, nor is it made up of particles.* These are merely models, analogies, mental images that help us to understand how light behaves in certain circumstances. In particular, since light is not really a wave or a

particle, as we understand them in our everyday experience, it is pointless to try to visualize a process in which light waves are converted to photons before being absorbed.

You may find that an analogy between the wave–particle model of light and a cross-bred animal is quite a helpful one. An animal breeder will take the male animal of one breed and the female of another breed, and mate them to produce offspring that have some characteristics of each parent. For example Charollais bulls which are noted for their meat have been mated with Jersey cows which are noted for their milk yield to produce a breed of cattle which are good for both meat and milk. The cross-bred animal is neither Charollais nor Jersey, but some hybrid of the two. We can say that when it comes to meat production it is really like a Charollais, but for milk production it is like a Jersey. In a similar way our wave–particle model is a cross between a wave model and a particle model. For visualizing the way that light travels it is just like the wave model, but for visualizing the way that light is absorbed it is just like a particle model. The hybrid model contains the best features of the two models from which it was produced.

7.2 Objectives of Section 7

Having studied Section 7, you should now be able to do the following:

(a) Identify situations in which the wave model is the best description of the behaviour of light, and situations in which the particle model is the best description.

(b) Use the wave–particle model to solve simple problems which involve both the wave and particle natures of light and of other electromagnetic radiation.

To test your achievement of these Objectives, try the following SAQs.

SAQs 21–4 Four processes involving light are listed in the following four SAQs and each is broken down into a number of stages. Which of these stages are best visualized with a wave model and which with a photon model?

SAQ 21 Light is emitted from an electric lamp; it passes through air, through a glass prism, and through more air; it strikes a sheet of photographic film (which is eventually developed to produce an image).

SAQ 22 Sunlight travels through the earth's atmosphere; it is partially scattered by dust.

SAQ 23 Sunlight reaches a leaf; it causes photosynthesis to occur.

SAQ 24 Light falls on this page; it is partially reflected; it travels to your eye; it produces the sensation of vision when it strikes the retina.

SAQ 25 Figure 44 shows an arrangement of two photocells P and Q, a double slit and a small monochromatic light source. The threshold frequency of the photocells is 6.0×10^{14} Hz, the spacing of the double slits is 4.0×10^{-6} m, and the wavelength emitted by the light source is 4.0×10^{-7} m. Use the wave–particle model to determine whether currents are produced by photocell P and/or photocell Q.

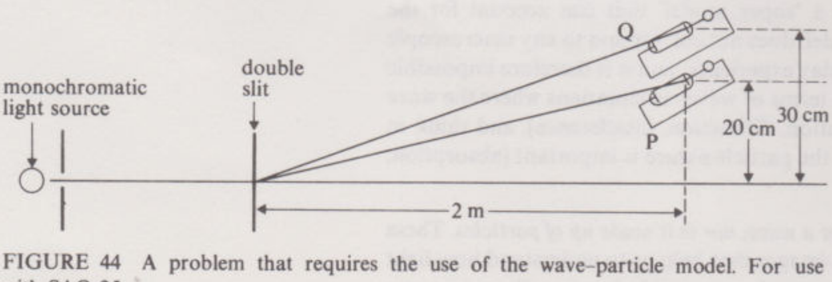


FIGURE 44 A problem that requires the use of the wave–particle model. For use with SAQ 25.

Objectives

For your convenience, the following list gathers together the Objectives that have been stated at various points throughout this Unit. It is a summary of the things that you should be able to do after studying this Unit.

1 Define correctly, recognize the best definitions of, and distinguish between true and false statements concerning the terms, concepts and principles listed in the third column of Table A.

2 Explain how, and to what extent, the six main properties of light discussed in the Audio-vision sequence (namely, straight-line propagation, decrease of intensity with distance, crossing of light from different sources, reflection, refraction, dispersion) can be accounted for by a wave model and by a particle model (SAQs 1 and 2).

3 Contrast the *predictions* made by wave and particle models (on the basis of refraction measurements) for the ratio of the velocity of light in two materials, and explain why *measurements* of the velocity of light in various media cause us to prefer a wave model to a particle model (SAQs 2 and 3).

4 Explain the meaning of the following parameters when used in connection with waves: wavelength, amplitude, intensity, period, frequency, velocity. Recall the relations between these parameters, $v = f\lambda$, $f = 1/T$, $I \propto A^2$, use these relations to calculate certain of these parameters when the values of others are specified, and relate these parameters to graphical representations of waves (SAQs 5, 6, 7, 11 and 12).

5 Explain what is meant by the principle of superposition, and apply this principle, together with the concepts of constructive and destructive interference, to simple diffraction problems (SAQs 8 and 9).

6 Describe the appearance of the diffraction patterns produced by a double slit, a diffraction grating and a single slit (Home Experiments 1–4).

7 Recall the condition for constructive interference with a double slit or a diffraction grating ($\sin\theta = n\lambda/d$), and use this condition in various problems to calculate one of the parameters θ, n, λ, or d, when given the values of the other parameters (SAQs 10 and 25).

8 Describe a simple method of measuring the wavelength of light (Home Experiment 2).

9 Explain why diffraction patterns are not observed with all light sources.

10 Explain what is meant by an electromagnetic wave (SAQ 12).

11 Give reasons that support the conclusion that light is an electromagnetic wave.

12 Place in order the various regions of the electromagnetic spectrum according to their frequency (or wavelength), and recall the orders of magnitude of the frequency (or wavelength) of the waves in the different regions (SAQs 11, 12, 13 and 19).

13 Recall that all electromagnetic waves travel at a velocity of $3 \times 10^8\,\mathrm{m\,s^{-1}}$ in air (SAQ 11).

14 Describe photoelectric experiments and their results, and explain to what extent they can be accounted for by the wave model and by the photon model. (SAQ 20).

15 Perform simple calculations involving (a) the threshold frequency of photocells, (b) the conversion of photon energy to electron kinetic energy, (c) the conversion of kinetic energy to electrical energy (SAQs 14, 15, 16, 17 and 25).

16 Recall that photon energy $E = hf$, and calculate the energy when given the frequency, or vice versa (SAQs 17, 18 and 19).

17 Recall how the intensity is related to the number of photons, and calculate one of these quantities when given the other (SAQ 18).

18 Identify situations in which the wave model is the best description of the

behaviour of light, and situations in which the particle model is the best description (SAQs 21–24).

19 Use the wave–particle model to solve simple problems which involve both the wave and particle natures of light and other electromagnetic radiation (SAQ 25).

Appendix 1: Comments on the Home Experiments

Home Experiment 1

We expect that you observed the following effects. The light appears to come not only from the light bulb but also from a number of regions on either side of the slit that is in front of the bulb. With the closest pair of slits (spacing 0.08 mm) there are two bright regions on either side of the light bulb, together with other fainter regions. With the 0.16 mm pair there are about five bright regions on either side. It is very difficult to count how many bright regions there are for the most widely spaced slits, but there are about twice as many as observed with the medium-spaced slits. For all three double slits, the total angle over which all of the beams are spread is about the same.

If you did *not* observe these effects, check the following points and then try again:

(a) The slit in front of light bulb should be about *1 mm wide.*

(b) The double slit should be *parallel* to the slit in front of the light bulb, i.e. it should be vertical.

(c) The double slit should be held about *one metre* from the light bulb, *as close as possible* to the eye, and moved until you can see the light bulb. If you wear spectacles, there is no need to remove them.

There are obvious similarities between what you observed in this experiment and the water wave experiment shown in Figure 4 (p. 11). Light is spread out, diffracted, into a number of fairly well-defined directions. The dark areas midway between these directions are places where the light is not apparent, and these areas correspond to the regions where no water waves are seen. The light is split up into a larger number of directions than the water waves, but these directions are much closer together than with water.

The effect of changing the slit spacing is similar in both cases: doubling the spacing between the slits causes the angles between the different directions in which light/water is diffracted to decrease by about a factor of two.

Home Experiment 2

If you used the pair of slits with the closest spacing (0.08 mm) then you should have observed five strong beams of light (including the beam coming directly from the bulb), and a number of fainter beams. When one of the Course Team members tried out the experiment, he got the following results:

Third beam to left of centre:	3.4 cm on scale, ± 0.2 cm
Third beam to right of centre:	7.0 cm on scale, ± 0.2 cm
difference =	3.6 cm $\pm$ 0.3 cm

(The error in the difference is unlikely to be as large as 0.4 cm, the sum of the errors in the measured positions. A reasonable estimate is ± 0.3 cm.)

So the distance between third beam and centre = 3.6/2 = 1.8 cm $\pm$ 0.2 cm

Distance between double slit and scale = 95 cm $\pm$ 1 cm

Spacing d of double slit = 0.080 mm $\pm$ 3 per cent

For third beam, $n = 3$

To find a value for the wavelength of light, he used equation 6: $\sin\theta = n\lambda/d$. This can be rearranged into a more convenient form, namely

$$\lambda = \frac{d\sin\theta}{n}$$

Now $\sin\theta$ = opposite/hypotenuse, by definition, and the way that this relates to this experiment is shown in Figure 26 (p. 26):

$$\begin{aligned}\text{opposite} &\equiv \text{measured scale distance between third beam and central beam}\\ \text{hypotenuse} &\equiv \text{distance between slit and scale}\end{aligned}$$

(Note that since the angle θ is so small, it makes no difference whether you measured the distance from the double slit to the centre of the scale, or to the point on the scale from which the third beam appeared to come—these lengths only differ by a fraction of a millimetre.)

So, for the results quoted above,

$$\sin\theta = 1.8\,\text{cm}/95\,\text{cm}$$

and therefore

$$\lambda = \frac{d\sin\theta}{n} = \frac{(0.080 \times 10^{-3}\,\text{m}) \times 1.8\,\text{cm}}{3 \times 95\,\text{cm}}$$

$$\lambda = 5.1 \times 10^{-7}\,\text{m}$$

What about the possible error in this value of the wavelength? Well, the fractional error in d is 0.03 (that is, 3 per cent), the fractional error in the distance between third beam and centre is $0.2/1.8 = 0.11$, and the fractional error in the distance between scale and slits is $1/95 = 0.01$. Since the first and third of these errors are much smaller than the second, they can be neglected. Therefore the fractional error in the wavelength is 0.11, so:

$$\lambda = (5.1 \pm 0.6) \times 10^{-7}\,\text{m}$$

If you got a result similar to this, then award yourself a well-earned pat on the back! If you were unsuccessful in your first attempt, either because you were unsure of what to measure or because you were unsure of how to do the calculations, then have another shot. After all, it is not every day that you measure such a tiny length with such simple apparatus!

Home Experiment 3

You should have observed a white line in the position of the light bulb, and several rainbows of colour on either side. Now as the earlier discussion of diffraction by a grating showed, and as was indicated in Figures 29b and 29c, light with a certain wavelength λ is diffracted only at very well-defined angles that satisfy the equation $\sin\theta = n\lambda/d$. The fact that you observed different coloured light at different angles should therefore indicate to you that the different colours of the rainbow are associated with waves that have different wavelengths. In addition, since the blue colours appear at smaller angles than the red, you can conclude that:

$$\lambda_{\text{blue}} < \lambda_{\text{red}}$$

'White' light is, in fact, a mixture of waves, with wavelengths that range from about 4×10^{-7} m for blue light to about 7×10^{-7} m for red light. If you wish, you can measure the wavelengths for yourself by using a similar technique with the grating to that used in Experiment 2 with the double slit.

Home Experiment 4

The kind of pattern that you should have observed is broad areas of light on either side of the slit in front of the light bulb, flanked by dark areas and then by fainter areas of light. If you did the experiment in a very dark room, you may have seen additional light and dark areas.

As with the double slit and the diffraction grating, the dark areas are regions of destructive interference. However, in the case of the single slit it is waves from different parts of the *same* slit that are interfering rather than waves from different slits, and the angles at which destructive interference occurs are related to the width of the slit.

SAQ answers and comments

SAQ 1 (i) All of the properties listed in the Table can be explained by both wave *and* particle models, except for partial reflection and partial refraction at an interface between two transparent materials; this cannot be explained with a simple particle model.

(ii) None.

(iii) The fact that light usually appears to travel in straight lines, and casts shadows, can be modelled by seismic waves but apparently not by water waves or sound.

SAQ 2 Wave and particle models make *opposite* predictions for the ratio of the *velocity* of light in two different materials. These predictions are based on experimental observations of the *refraction* of light and of waves and particles.

SAQ 3 (a) The wave model predicts that the velocity is higher in the oil, since the angle of incidence in the oil is greater than the angle of refraction in the water.

(b) The particle model makes the opposite prediction, namely that the velocity is higher in the water.

(c) The measured velocity would be higher in the oil. Experiments have confirmed that the predictions of the wave model are correct.

SAQ 4 $2.06 \times 10^8\,\mathrm{m\,s^{-1}}$. The velocity is inversely proportional to beam displacement (equation 1). Thus the velocity in glass B is given by

$$\frac{c_B}{c_A} = \frac{\text{displacement with glass A}}{\text{displacement with glass B}}$$

Hence $$\frac{c_B}{2.00 \times 10^8\,\mathrm{m\,s^{-1}}} = \frac{10.3\,\mathrm{mm}}{10.0\,\mathrm{mm}}$$

$$c_B = 2.06 \times 10^8\,\mathrm{m\,s^{-1}}$$

SAQ 5 Wavelength $\lambda = 50\,\mathrm{mm} = 0.05\,\mathrm{m}$; this is the distance between corresponding points on the wave profile in Figure 13a, such as the distance between crests, or the distance between troughs.

Period $T = 0.4\,\mathrm{s}$; this is the distance between corresponding points on the wave profile in Figure 13b. Note that the period is the 'repeat' *time* on the graph that shows the wave as a function of time, whereas the wavelength is the 'repeat' *distance* on the graph that shows the wave as a function of position.

Frequency $f = 2.5\,\mathrm{Hz}$; this is the reciprocal of the period.

Velocity $v = 0.125\,\mathrm{m\,s^{-1}}$; this is found by using equation 4, $v = f\lambda$, or equation 2, $v = \lambda/T$.

Amplitude $A = 5\,\mathrm{mm}$; this is the difference between average height and crest (or trough) height.

SAQ 6 4 : 1. The intensity is proportional to the square of the amplitude. The ratio of the amplitudes is 2 : 1, and so the ratio of the intensities is $2^2 : 1 = 4 : 1$.

SAQ 7 The correct graphs are shown in Figure 45. In one quarter of a period $T/4$ the wave moves one quarter of a wavelength $\lambda/4$ to the right; in $T/2$ it moves $\lambda/2$, and so there is then a trough where each crest used to be; in one period T the wave moves one wavelength and it therefore cannot be distinguished from its initial position; in two periods it moves two wavelengths and so it is again the same as at the start.

SAQ 8 The total displacement is shown by the thick line in Figure 46. This displacement curve was obtained by adding the displacements of the two waves represented by the thinner lines.

SAQ 9 A, D constructive; C destructive; B intermediate. Constructive interference occurs at positions where the difference between the distances to the two slits is an integral number of wavelengths $n\lambda$; destructive interference occurs where the difference is $(n + \frac{1}{2})\lambda$; and intermediate type of interference will occur when the difference has some other value. The wavelength is 10 mm, that is, the distance between wavefronts. A is a position

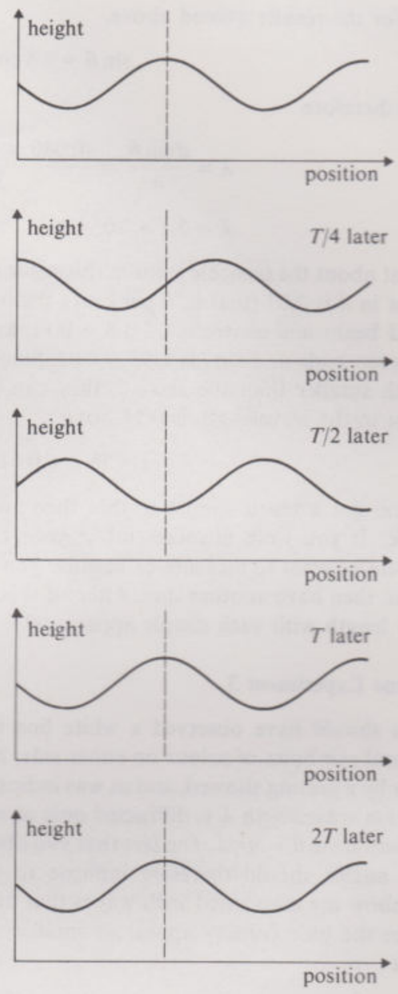


FIGURE 45 The answer to SAQ 7.

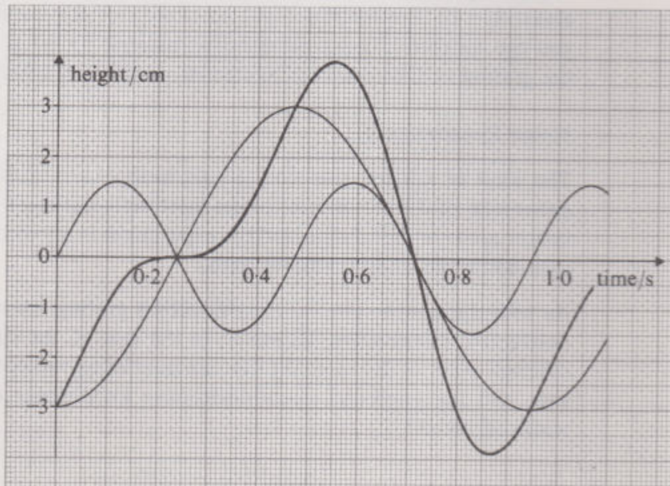


FIGURE 46 The answer to SAQ 8.

where constructive interference occurs since the difference between the distances to the two slits is $(82 - 72) = 10\,\text{mm} = \lambda$. For D the difference is $20\,\text{mm} = 2\lambda$, so this is also a position of constructive interference. At C destructive interference occurs since the difference is $5\,\text{mm} = \frac{1}{2}\lambda$. At B the interference is neither constructive or destructive since the difference is $3\,\text{mm} = 0.3\lambda$.

SAQ 10 0 degrees, 17 degrees, 35 degrees and 59 degrees. The strong beams (constructive interference) occur when $\sin\theta = n\lambda/d$ (equation 6). Since $\lambda = 20\,\text{mm}$ and $d = 70\,\text{mm}$, the possible values of $\sin\theta$ for different values of n are:

$$\sin\theta = 0,\ 1 \times (20/70),\ 2 \times (20/70),\ 3 \times (20/70)$$

$$\sin\theta = 0,\ 0.29,\ 0.57,\ 0.86$$

Using a calculator or tables, you can find that the angles which have these values for $\sin\theta$ are:

$$\theta = 0,\ 17\text{ degrees},\ 35\text{ degrees},\ 59\text{ degrees}$$

Since there is no angle that has a sine that is greater than unity, no other beams are found. For example, if $n = 4$, then

$$n\lambda/d = 4 \times 20/70 = 1.14$$

and no angle θ can have $\sin\theta = 1.14$.

SAQ 11 No. This should be obvious from the fact that the values quoted for wavelength and frequency do not lie opposite each other on the scales in Figure 35. In fact the velocity of these rays is $v = f\lambda = (3 \times 10^{22}\,\text{Hz}) \times (10^{-11}\,\text{m}) = 3 \times 10^{11}\,\text{m s}^{-1}$. This is a thousand times faster than the velocity of electromagnetic waves! They are a figment of the author's imagination. If the frequency had been $3 \times 10^{19}\,\text{Hz}$, then the velocity would have been $3 \times 10^{8}\,\text{m s}^{-1}$, and the weapon would have been an X-ray gun.

SAQ 12 Your sketches should be similar to Figures 33 and 34 except for the magnitudes of wavelength and frequency. A typical wavelength for ultraviolet waves is $10^{-7}\,\text{m}$ and the corresponding frequency is $3 \times 10^{15}\,\text{Hz}$.

SAQ 13 (i) Microwave, (ii) γ-ray, (iii) visible light, (iv) infra-red; you can check these answers by referring to Figure 35. (v) $\lambda = 10^{-8}\,\text{m}$ is on the boundary between ultraviolet and X-rays. This boundary is not clearly defined (nor are the other boundaries) and sometimes waves in this region are known as short-wavelength ultraviolet, sometimes as long-wavelength X-rays.

SAQ 14 (a) and (b) will produce photoelectric currents since the frequencies are greater than the threshold frequency f_t. (c) and (d) will not produce any current.

SAQ 15 The positively-charged ball will strike the negative plate with a velocity of $0.1\,\text{m s}^{-1}$. The change in electrical energy of the ball (QV) is converted to kinetic energy ($\frac{1}{2}mv^2$). Thus $QV = \frac{1}{2}mv^2$, or, multiplying both sides of this equation by $2/m$, $v^2 = 2QV/m$.

$$\text{So}\quad v^2 = \frac{2 \times (10^{-9}\,\text{coulomb}) \times (50\,\text{volts})}{(10^{-5}\,\text{kg})} = 10^{-2}(\text{m s}^{-1})^2$$

$$\text{or}\quad v = 0.1\,\text{m s}^{-1}$$

SAQ 16 The ball *will* strike the positive plate. Its electrical energy at the positive plate is higher by the amount $QV = (10^{-9}\,\text{C}) \times (50\,\text{V}) = 5 \times 10^{-8}\,\text{J}$ than it is halfway between the plates. Its initial kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2} \times 10^{-5} \times (0.2)^2 = 2 \times 10^{-7}\,\text{J}$.

This is more than enough to provide the electrical energy at the positive plate.

SAQ 17 $6.6 \times 10^{-20}\,\text{J}$.

The *minimum* amount of energy required to remove an electron is $E_b = hf_t = h \times (5 \times 10^{14}\,\text{Hz})$. The photons have energy $h \times (6 \times 10^{14}\,\text{Hz})$, and so the excess energy of the photon, that is, $h \times (1 \times 10^{14}\,\text{Hz})$, is converted to kinetic energy. Obviously the smaller the energy used to remove the electron, the greater will be the energy left over for conversion to kinetic energy. Thus the maximum kinetic energy is:

$$h \times (10^{14}\,\text{Hz}) = (6.63 \times 10^{-34}\,\text{Js}) \times (10^{14}\,\text{Hz}) = 6.63 \times 10^{-20}\,\text{J}$$

SAQ 18 2.5×10^{20} photons.

Each photon has energy $hf = (6.63 \times 10^{-34}\,\text{Js}) \times (6 \times 10^{14}\,\text{Hz}) = 4 \times 10^{-19}\,\text{J}$. The intensity is $10^3\,\text{watt m}^{-2}$, which is the same as $10^3\,\text{joule m}^{-2}\,\text{s}^{-1}$, so the number of photons per square metre per second is $N = I/hf = 10^3/(4 \times 10^{-19}) = 2.5 \times 10^{21}$. The number striking this Unit would be $0.1 \times (2.5 \times 10^{21}) = 2.5 \times 10^{20}$. There are plenty of them!

SAQ 19 From Figure 35 you can see that a typical infrared frequency is $3 \times 10^{13}\,\text{Hz}$, and this leads to an energy of $hf = 2 \times 10^{-20}\,\text{J}$.

SAQ 20 (a) Photoelectric current (that is, the rate of ejection of electrons) depends on frequency; no current is observed if the light frequency is below a certain critical value. The wave model predicts that current depends on intensity, but not on frequency.

(b) The maximum energy of the photoelectrons increases linearly with frequency. The wave model would predict that the energy of the electrons was independent of frequency.

(c) Photoelectrons are ejected instantaneously even when the light intensity is low. The wave model predicts that when the intensity is low it would take an appreciable time to transfer enough energy to an electron to remove it from the metal.

SAQ 21 Light emission by the lamp and absorption by the film are best visualized with the photon model. The passage of the light through air and its refraction and dispersion into its different colours by the prism are best visualized with the wave model. Remember that waves are best for propagation, and photons are best for emission and absorption.

SAQ 22 The wave model explains both the travelling and the scattering. The latter is essentially diffraction of the waves by the small dust particles.

SAQ 23 The wave model describes how the light reaches the leaf, and the photon model describes how it produces the chemical reaction known as photosynthesis.

SAQ 24 The way that the light travels and the way that it is reflected are described by the wave model. Its interaction with the retina is described by the photon model. The photon model also describes the absorption of light by the page.

SAQ 25 Photocell P will produce a current but photocell Q will not. To work out why this is so, we first use the wave model to determine where constructive interference occurs, from $\sin\theta = n\lambda/d$. This tells us that $\sin\theta = n \times (4 \times 10^{-7}\,\text{m})/(4 \times 10^{-6}\,\text{m}) = 0.1\,n$, and hence $\theta = 0$, 0.1 radians, 0.2 radians, 0.3 radians, and so on. (Remember that $\sin\theta = \theta$ radians, when the angle is small.) The angle between photocell P and the normal to the slits is $\theta_P = 0.2\,\text{m}/2\,\text{m} = 0.1$ radians, and this photocell is therefore in a position of constructive interference. Photocell Q is at an angle $0.3\,\text{m}/2\,\text{m} = 0.15$ radians, which is a position of destructive interference. So the wave model indicates that light will fall on photocell P but not on photocell Q. It is clear that photocell Q cannot produce any current.

To determine whether photocell P produces any current we need to use the photon model. This tells us that a current will be produced if the frequency of the light is greater than the threshold frequency. The frequency of the light is

$$f = c/\lambda = (3 \times 10^{8}\,\text{m s}^{-1})/(4 \times 10^{-7}\,\text{m}) = 7.5 \times 10^{14}\,\text{Hz}.$$

Since this is above the threshold frequency ($6.0 \times 10^{14}\,\text{Hz}$) we can conclude that current will be produced by P.

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Figure 3a is reproduced courtesy of the Coastal Engineering Research Center, U.S. Department of the Army.

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